Politecnico di Milano

Radiation Detection Systems

Hand Notes on the course programme

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a.a. 2023-2024

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***Disclaimer**: these are my hand notes on the course of Radiation Detection Systems, taken during the academic year 2023-2024. I made these notes in preparation for the exam, reviewing the theory using mainly the slides. As they are not based on live class notes, some observations made by the professor may be missing.*

These notes are not a substitute of the slides. Especially on the last part, some slides content may be missing. I made these notes for myself and did not plan to publish them in advance, so I also apologize for my bad writing and page layout.

*Last thing, don't take everything in these notes as true! Some uncertainties are still in it, and maybe even errors I'm not aware of! So please, **question everything** and check yourself if what is written makes sense. I personally suggest in following the course as the professor for me was really clear and also available to discuss every doubt.*

Good luck!

MEGASCHEMA RADIATION

Monday, April 29, 2024 10:42 AM

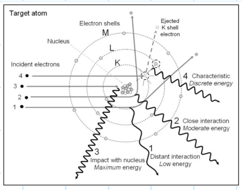
RADIATION SOURCES

$E(eV) = \frac{1240}{\lambda(nm)}$ $E = h\nu = \frac{hc}{\lambda}$

$\uparrow$ frequency

Xrays : some eVs

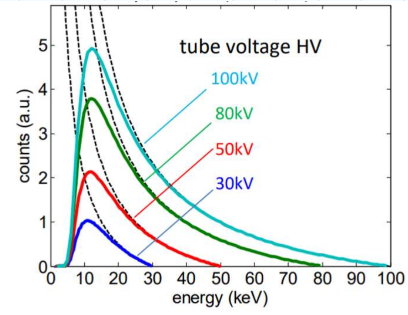
ENERGY



Hitting material with electrons

=> curving electrons => accelerating electrons

=> release of photons



spectrum of electrons including characteristic peaks (XRF)

ph/λ

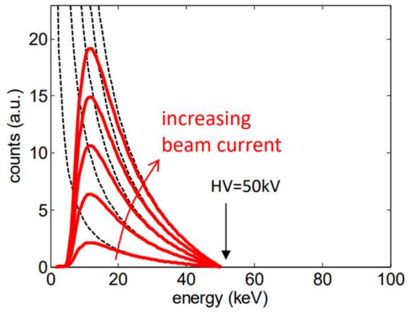
$$I(\lambda) = \frac{K}{\lambda^2}\left(\frac{\lambda}{\lambda_{min}} - 1\right)$$

$\uparrow$

[ph/λ] radiation intensity $= \frac{dn}{d\lambda}$

$\lambda_{min} \Rightarrow E_{max} = V\cancel{/}q$ $V \cdot q$

E_{MAX} proportional to high volage giving energy to electron



ph/E

$$I(E) = \left|\frac{dn}{d\lambda}\right| \cdot \frac{d\lambda}{dE} = \frac{K}{hc}\left(\frac{E_{max}}{E} - 1\right)$$

[ph/E(eV)]

$\lambda = \frac{hc}{E}$

$E = \frac{hc}{\lambda}$

$\left|\frac{d\lambda}{dE}\right| = hc\frac{1}{E^2}$

E_{ph}/E

$$\mathcal{E}(E) \cdot I(E) \cdot E = \frac{K}{\lambda}\left(\frac{E_{max}}{E} - 1\right)$$

$\uparrow$ emitted energy

$\frac{hc}{\lambda}$

$(E_{phout}/E(eV))$

$\uparrow$

$[ph_n \cdot E_{ph}]$

$$= \frac{K}{hc} \cdot (E_{max} - E)$$

$$= \bar{K}(E_{max} - E)$$

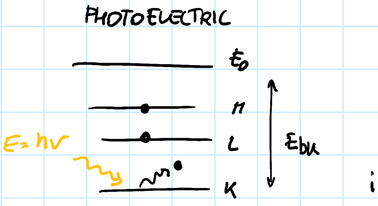
BREMMSTRAHLUNG X-RAYS

Other radioactive sources ..

- × synchronous radiation
  - ↪ circular movement of electrons
- × Radioactive dement decay

RADIATION SOURCES

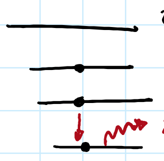
PHOTONS INTERACTION



$e_{out} = E - E_{bK}$

inner shells
atom

XRF

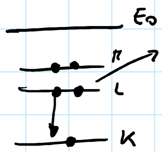


$XRF_{ph} = E_{bK} - E_{bL} = E_{K\alpha}$

a photon is relased with the excess energy gained from the decay L→K

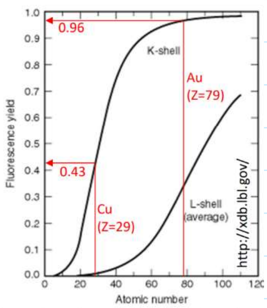
=> photon energy characteristic of material

AUGER



$(E_{bK} - E_{bL}) - E_{BL}$
↑
(respect to E_0)

another electron is released with excess Kinetic energy from the decay of the second electron

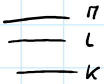


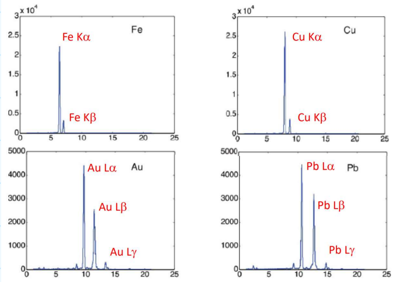
Characteristic X-rays derive from XRF

=> fluoroscence yield
probability of XRF agains Auger

=> heavier elements tends to XRF more
(Auger is harder in heavier elements, becuse meeds another extraction from an inner shell)

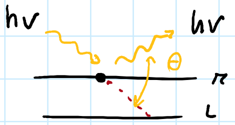
XRF nomenclature	K_α :	drop from L (α = 1 above) to K
	K_β	drop from M (β = 2 above) to K
	L_α :	drop from M (α = 1above) to L





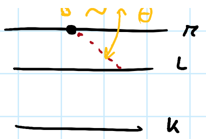
possible to do spectroscopy

PHOTOELECTRIC EFFECT



- elastic scattering
  => no energy loss
- electron vibrates and

$x = \sin(\theta)/\lambda$
↑
momentum transfer

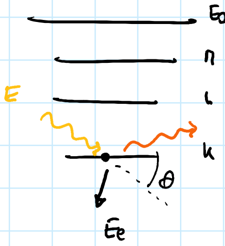


=> no energy loss

- electron vibrates and re-emits the photon
- change of momentum as direction changes

momentum transfer

RAILEIGH SCATTERING



- inolastic scattering
  => photon loses energy

- relevant only at high energies

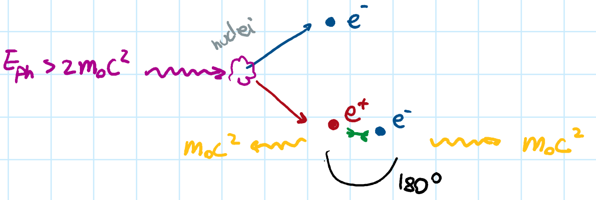
$$E' = \frac{E}{1 + \frac{E}{mc^2}(1-\cos\theta)}$$

$E \gg m_0c^2$
511 keV

$$E_e = E - E' - E_K$$

↑ ???

COMPTON EFFECT



- interaction with nucleous
- extrimely HE photon (> 2 rest mass electron)
- e-e^+ (positron) pair production
- positron soon annichilates w other electrons
  => producing 2 photons

PAIR PRODUCTION

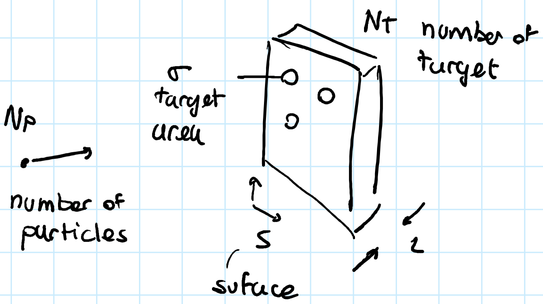
INTERACTION PROBABILITY

$$P_{int} = \frac{N_t\sigma}{S} = \frac{n_t \cdot \sigma S L}{S} \quad [1]$$

interaction probability

$$= n_t \sigma L \quad [1]$$

↑
target/cm^3 density



L ≪ attenuation lenght

$$n_T = \frac{\rho}{Au}$$

ρ = g/cm^3 density
A = atomic mass
u : atoic mass unit

$n_t = \frac{}{Au}$ A : atomic mass

u : atoic mass unit

$$P_{int} = \frac{\rho}{Au}\sigma L = \underset{\uparrow}{\mu}\rho L$$

$$\mu = \frac{\sigma}{Au} \quad [cm^2/g]$$ mass attenuaction coefficient

INTERACTION PROBABILITY

$$\Phi \ [ph/A \cdot t] = \frac{N_p}{S \cdot \Delta t}$$

↑ incident flux densiy

(und rate) ?

$$N_{events} = N_{ph} \cdot P_{int} = \Phi S \Delta t \frac{N_t \sigma}{S}$$

$$= \Phi N_t \sigma \Delta t$$

$$R_{events} = \frac{N_{events}}{\Delta t} = \Phi N_t \sigma$$

cross section is founde and defined experimentaly from ↑

$$\sigma = \frac{N_{events}}{\Phi N_t \Delta t} = \frac{N_{events}}{N_{ph}} \cdot \frac{S}{N_t} = \frac{N_{ev}}{N_{ph} n_t L} \qquad N_t = n_t \cdot SL$$

EVENT RATE

For different events, we define different target cross section

=> the total cross sections takes all possible interactions into account

$$\sigma = \sigma_{Photoelectric} + \underset{\substack{\uparrow \\ \text{incoherent} \\ \text{(inlelastic)} \\ \text{scattering}}}{\sigma_{incoh}} + \underset{\substack{\uparrow \\ \text{elastic} \\ \text{(choerent)} \\ \text{scattering}}}{\sigma_{coh}} + \dots$$

derives

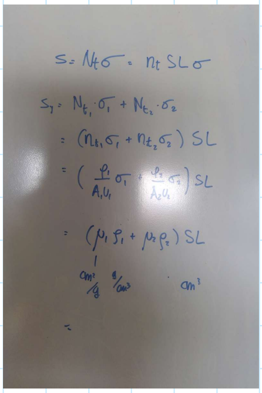
$$\Rightarrow \quad \mu = \mu_{pe} + \mu_{inch} + \mu_{coh} \dots$$

For mixed materials / compounds

$$\mu = \sum_i \underset{\uparrow}{w_i} \mu_i \quad [cm^2/g]$$

relative MASS weight →

in the compound



$$S = N_t\sigma = n_t SL\sigma$$

$$S_y = N_{t_1} \cdot \sigma_1 + N_{t_2} \cdot \sigma_2$$

$$= (n_{t_1}\sigma_1 + n_{t_2}\sigma_2) SL$$

$$= \left(\frac{\rho_1}{A_1 U_1}\sigma_1 + \frac{\rho_2}{A_2 U_2}\sigma_2\right) SL$$

$$= (\mu_1 \rho_1 + \mu_2 \rho_2) SL$$

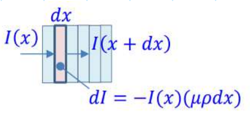
cm^2/g g/cm^3 cm^3

COMPOSITE MASS ATTENUATION COEFFICIENTS

$$P_{int} = \mu \rho L \quad \Rightarrow \quad p_{int} = \mu\rho \quad [cm^{-1}]$$

probability per unit lenght

probability per unit lenght



$$I(x+dx) - I(x) = -I(x)\,\mu_{int}\,dx \qquad \frac{dI}{dx} = -\mu\rho I(x)$$

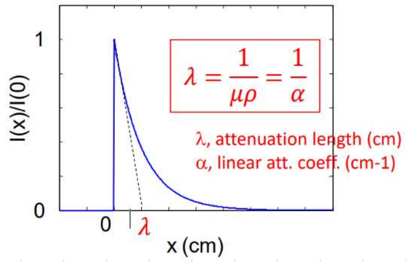
$$\int_{I_0}^{I(x)} \frac{dI}{I(x)} = \int_0^x - I(x)\,\mu\rho\,dx$$

$$\hookrightarrow \quad \Big[\ln(I)\Big]_{I_0}^{I(x)} = -\mu\rho x$$

$$\ln\left(\frac{I(x)}{I_0}\right) = -\mu\rho x$$

$$I(x) = I_0 e^{-\mu\rho x}$$

$$I(x) = I_0^{-x/\lambda}$$



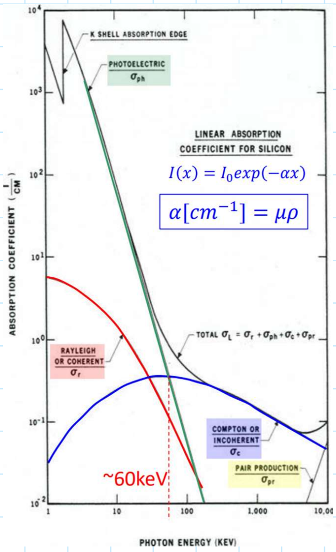
$$\lambda = \frac{1}{\mu\rho} = \frac{1}{\alpha}$$

λ, attenuation length (cm)
α, linear att. coeff. (cm-1)

λ [cm] attenuation lenght $= \frac{1}{\mu\rho} = \frac{1}{\alpha}$

α [cm^{-1}] $= \mu_{int} = \mu\rho$
attenuation coefficient

ATTENUATION LENGHT



$$I(x) = I_0 exp(-\alpha x)$$

$$\boxed{\alpha[cm^{-1}] = \mu\rho}$$

$\alpha = \frac{1}{\lambda}$ absorbtion coefficient

- dependence on photon energy
- different types of interaction

=> note the photoelectric K-shell jump when becomes

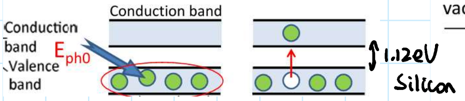
$$E_{ph} = h\nu > E_K$$

INTERACTION COMPONENTS

CHARGE GENERATION

Electron devices detect charge, so we must relate the photon interaction to a charge effect

VISIBLE - IR - UV light ~ 1 eV

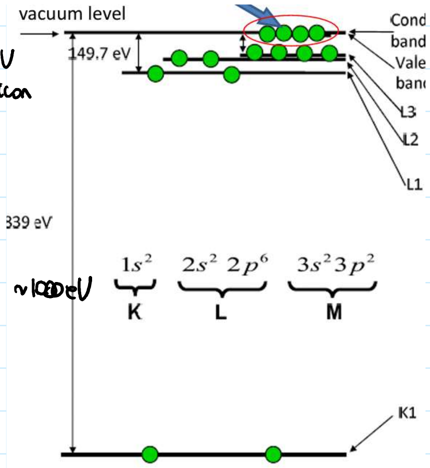


with low energy photons (visible light)

=> energy comparable to SEMICONDUCTOR BAND GAP

1 photon => 1 e-h generation

Charge independent from photon energy



E-H PAIR VISIBLE LIGHT

X RAY ~ 1000 eV

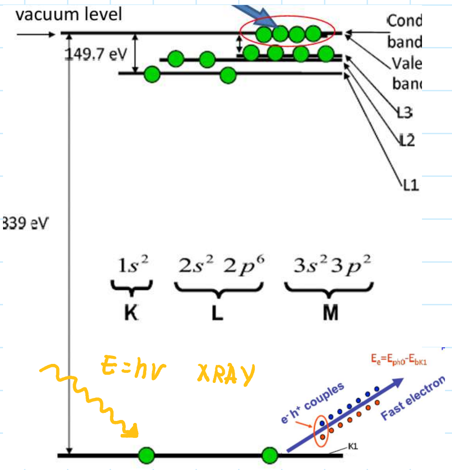
A x ray photon interacts with photoelectric effect with K shell ~ 1000 eV

=> The free electron has a high kinetic energy

=> The high kinetic energy

$E_{e\,free} = h\nu - E_{K\,shell}$

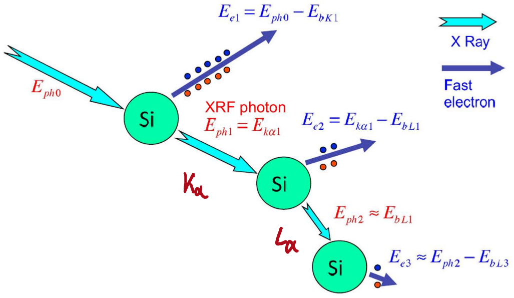
is dissipated through the creation of many e-h pairs (~1eV) in conduction / valence



charge dependant on photon energy

E-H PAIR GENERATION XRAYS

With high energy XRAYS a chain reaction can occour realising more XRF photons from different atoms in the materials



↳ Problem beause we will see XRF photns detected, but in rellity they are coming from the detector itself

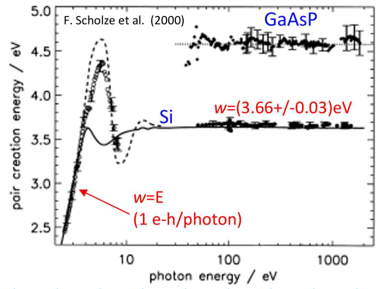
XRF CHAIN

The relation between radiation energy converted into e-h

The relation between radiation energy converted into e-h is a property of the material

$$w(E) = E/\langle N \rangle$$

$w = 3.63eV$ Si (300K)
$w = 2.96eV$ Ge (300K)



$\underbrace{\qquad\qquad}$ $\underbrace{\qquad\qquad\qquad\qquad}_{w \text{ constant}}$

$w(E) \propto E$ => $\langle N \rangle \propto E$

$\langle N \rangle$ constant

=> independent on photon E

LIGHT - UV - IR

XRAY

N number of generated e-h pairs

$$\langle N \rangle = \frac{E}{w(E)} = \frac{h\nu}{w(E)}$$

w constant for photoelectric XRAY generation

$$\sigma^2(N) = F \cdot \langle N \rangle = F \cdot \frac{E}{w}$$

F FANO FACTOR

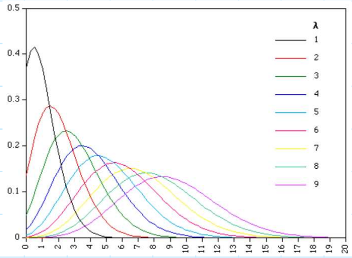
=> variance of generated e-h pair coefficient

$F = 0.12$ Si
$F = 0.13$ Si

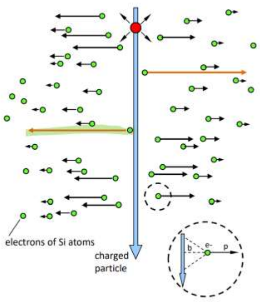
Where

N follows a poissonian (modified by fano factor, so $\sigma^2(N) \neq \langle N \rangle$) distribution

↳ tends to gaussian when $\langle N \rangle$ big



CHARGED PARTICLES INTERACTIONS



simplified model of passage of charged particle through matter (Coulomb scattering)

charged particles entering the device lose energy

- by giving kinetic energy to nearby electrons, ionizing the atoms ($\mu + \text{atom} = \mu + \text{atom}^+ + e^-$)
- by close collisions that generate gamma rays
- by emissions of a photon due to the deceleration induced by

through matter (Coulomb scattering)

- by emissions of a photon due to the deceleration induced by the substrate

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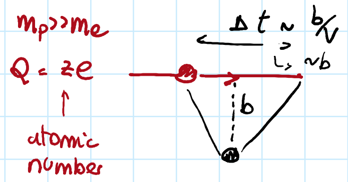
CHARGE INTERACTIONS



Assumptions

- Energy transfer >> binding energy
- Electron at rest during interaction (impulse transfer)
  ↳ whole momentum transfered to electron

Model:

$$F_\perp = \frac{1}{4\pi\varepsilon}\frac{ze^2}{b^2} \quad \text{coulomb force} \sim \frac{ze^2}{b^2}$$

$$\Delta t = \frac{b}{v} \quad \text{interaction time}$$

$$I_\perp = m v_e = \int F\,dt = \frac{ze^2}{b}\cdot\frac{1}{v} \quad \text{impulse = momentum transfer}$$
$$= p_e =$$

$$\hookrightarrow \quad v_e = \frac{ze^2}{b v m_e}$$

$$W = \frac{1}{2} m_e v^2 = \frac{p^2}{2m_e} = \frac{z^2 e^4}{b^2 v^2 2m_e}$$

I struggled on this part, so check!

↳ Energy transfer in a material

$$dE \overset{?}{=} \frac{z^2e^4}{b v^2 2m_e} \cdot \underbrace{v\,dt}_{dx} \Rightarrow \boxed{\frac{dE}{dx} = \frac{z^2e^4}{v^2 2m_e}\cdot\frac{1}{b}}$$

=> integrate over [bmin, bmax]

We then obtain <u>Bethe-Bloch formula</u>

↳ Energy loss is <u>normalized by target density</u>

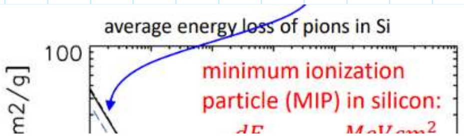
$$\underbrace{-\frac{1}{\rho}\frac{dE}{dx}}_{} \propto \frac{Zz^2}{A\beta^2} \propto \frac{1}{E} \propto \left[\frac{MeV}{cm}\,\frac{cm^3}{g}\right] \propto \frac{1}{E^2}$$

$\propto \frac{1}{v^2} \propto \frac{1}{E}$??

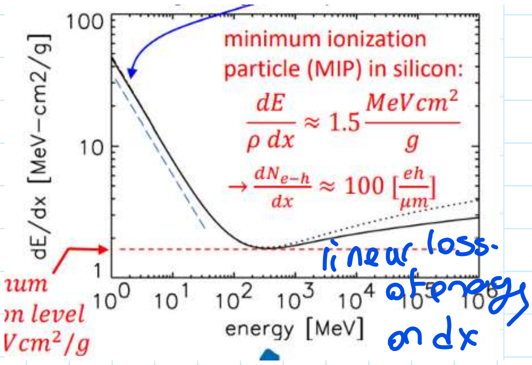
if plotted against energy

to normalize term with density

$v^2/c^2 \propto$ Energy of incoming particle (due to $1/v^2$ => velocity of ionizing incoming particle)



Using mean creation energy (W_0 in silicon)

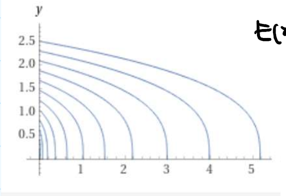


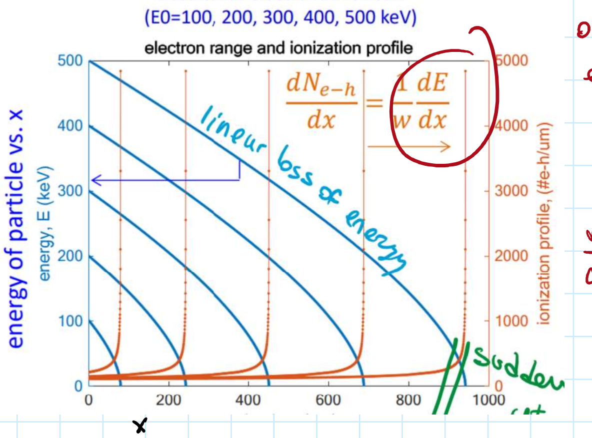
Using mean creation energy (W_e in silicon)
=> we can derive number of electron hole pairs generated with the energy released by the incoming radiation in silicon
=>

$$\frac{dN_{e-h}}{dx} = \frac{dN_{e\text{-}h}}{W_e\, dx} \approx 100\, e\text{-}h/\mu m$$

Ionization profile is obtained through integration

$$-\frac{dE}{dx} = f(E) \cancel{\propto \frac{1}{E^2}}$$ ⇐ differential equation



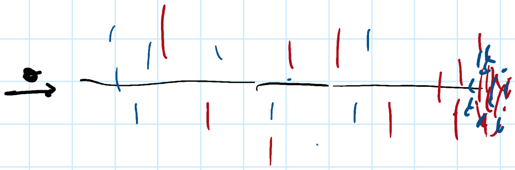


E tends to 0 near the end

$\frac{dE}{dx} \propto \frac{1}{E^2}$ ↓

$\frac{dE}{dx}$ ↑

Big energy released near the end!



trajectory with big drop of energy near the end

MEGASCHEMA SEMICONDUTORS

Monday, June 24, 2024 2:39 PM

1) PAIR CREATION ENERGY 2-5 eV (3.63eV Si)

- low : large number of charges in detector per photon
- higher energy resolution => high S/N ratio

2) ENERGY GAP 1- 3eV

- small leakage current (good medium high bandgap)
- ↳ less shot noise

3) DENSITY $\rho = 2\text{-}10\ g/cm^3$

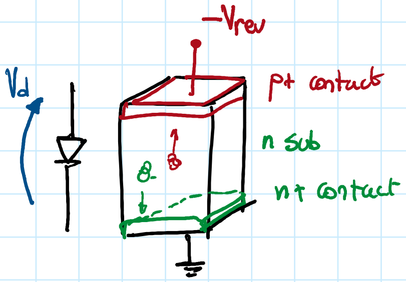
- high energy loss per unit lenght for ionized part. detection

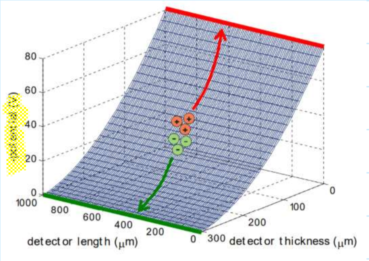
$$\frac{dN_{eh}}{dx} \approx 100\ eh/\mu m \quad Si \quad MIP$$

(minimum ionization particle)

- ↳ thin detectors
- ↳ precise position measurement

WHY SEMICONDUCTOR





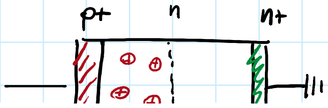
pn junction is reversed

$$C_{dep} = \varepsilon_0 \varepsilon_r \frac{A}{t_{dep}}$$

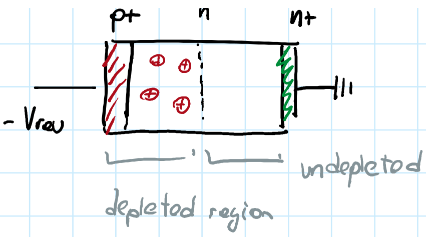
A ⟶ detector active area

t_{dep} ⟶ depletion region width

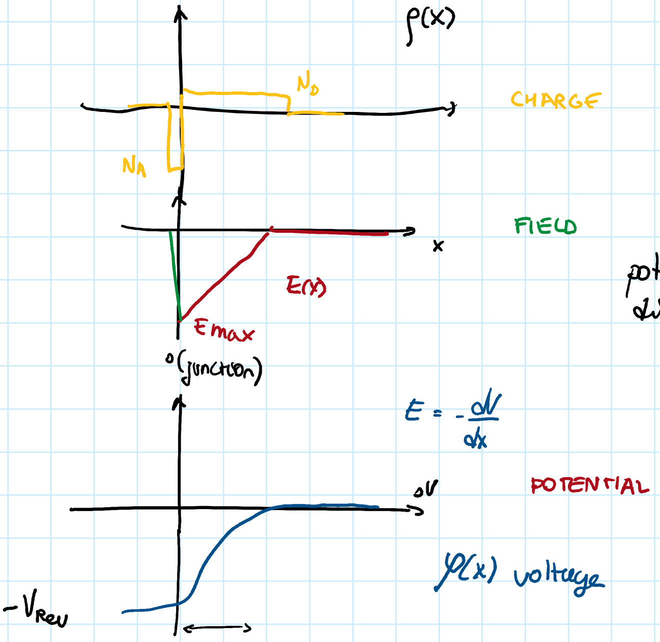
PN JUNCTION



Detectors pn junction are reverse-biased



Detectors pn junction are reverse - biased



$E = -\frac{dV}{dx}$

potential difference and electric field develops across depletion region width

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon} \qquad -\frac{d\varphi}{dx} = E$$

potential - field - charge equations 1D

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\varepsilon} \qquad \frac{d^2\varphi}{dx} = -\frac{qN_D}{\varepsilon}$$

poisson equation 1D

poisson equation

Assuming a unilateral Junction - highly doped p+

$$\rightarrow E(x) = \int_0^x \rho/\varepsilon \, dx = \frac{\rho}{\varepsilon}(x - x_d)$$

$$|E_{max}| = \frac{qN_d}{\varepsilon} x_{dep}$$

$x_{dep} \sim x_n$

$$V_{bias} = \int_0^{t_{dep}} -E dx = \sim \underbrace{\frac{1}{2}|E_{max}| x_d}_{\text{triangle area}} = \frac{1}{2}\frac{qN_D}{\varepsilon} x_d^2$$

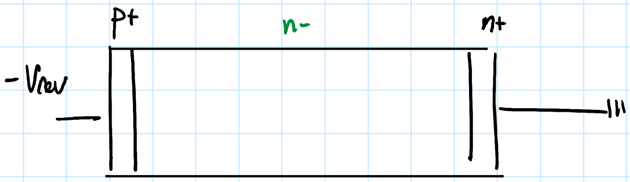
x detection is useful only if it happens in the depleted region

- x detection is useful only
  if it happens in the depleted region

  => field move charges and creates signal

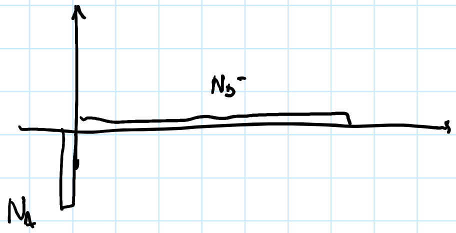
- x if undepleted, the generated electron-hole pair
  shortly recombines

PN JUNCTION DETECTOR

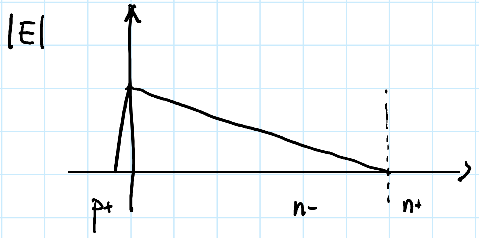


how to avoid
neutral region problems:
fully deplete region

how to fully deplete:
dope n region to have t_{dep} = width device

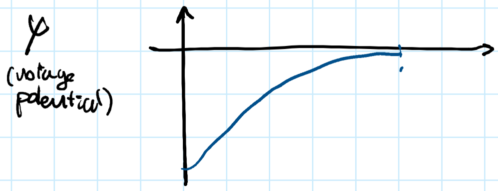


$$\rho_n = qN_D$$



$$\frac{dE}{dx} = \frac{qN_D}{\varepsilon}$$

$$|E_{max}| = \frac{qN_b}{\varepsilon} \cdot x_{dep} = \frac{qN_D}{\varepsilon} W$$

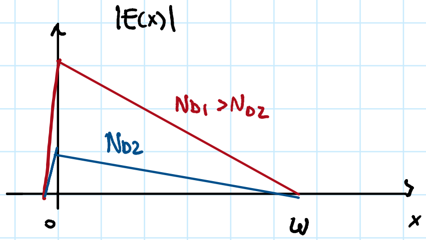


$$-\frac{d\varphi}{dx} = E$$

$$V_{dep} = \int -E\,dx = \frac{1}{2} \underbrace{\frac{qN_D}{\varepsilon} W}_{E_{max}} W^2$$

- x neutral region removed, so possible to detect
  charge in whole device
- x back side unwanted illumination is possible

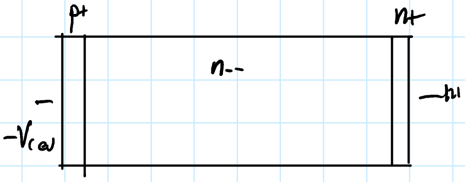
Balance between



$$E_{max} = \frac{qN_D}{\varepsilon} w \qquad \text{critical field} \propto \text{doping}$$

- need low substrate doping avoiding large E field

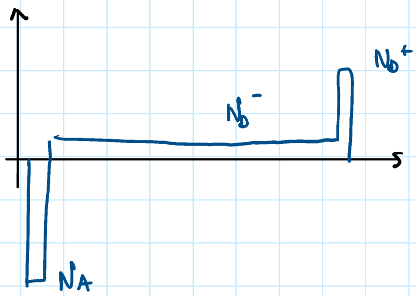
FULLY DEPLETED PN DETECTOR

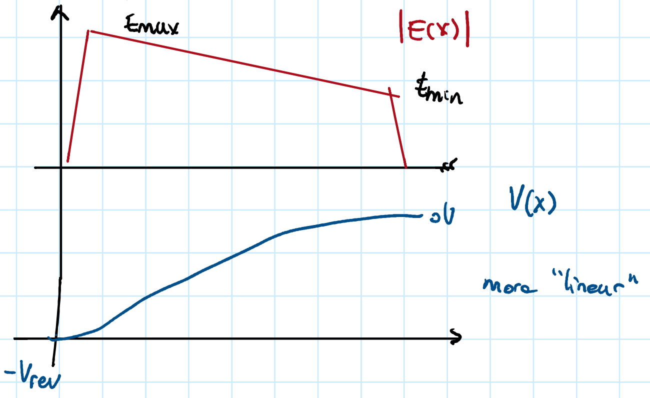


the fully depletion concept can be also improved: overdepleting - punch through of electric field

Doping in the n- region is so low that the electric field punches through to the n+ region

Depletion region reaches n+





$$V_{bias} = V_{dep} + \Delta V_{OD}$$

$$E_{min} = \frac{\Delta V_{OD}}{d} \quad \text{(offset of constant } E_{min}\text{)}$$

$$|E_{max}| = E_{min} + |E_{dep}|$$

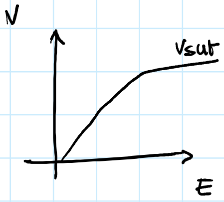
$$= \frac{\Delta V_{OD}}{d} + \frac{2V_{Dep}}{d}$$

↳ $V_{dep} = \frac{1}{2} E_{dep} d$

$$\underset{n}{E(x)} = E_{min} + \frac{qN_D}{\varepsilon}(x_0 - x)$$

× E_{min} helps avoiding near zero fields, resulting in a low velocity of generated e-h pairs and in a more probability of recombination x E_{min} could be sized to ensure that the carriers are always in saturation velocity

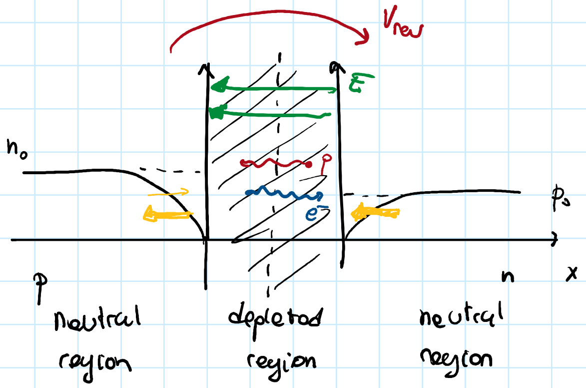
As Drude model



$$v = \mu E \qquad v_{sat} = \underset{\substack{\uparrow \\ \text{mobility}}}{\mu} E_{sat}$$

$$E_{min} = E_{sat} = \frac{v_{sat}}{\mu} \longrightarrow 10^7 \, cm/s \quad Si$$

PUNCH THROUGH - OVERDEPLETED PN JUNCTION DETECTOR



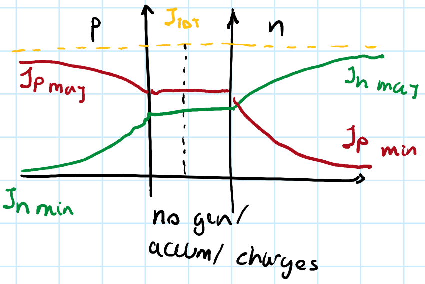
minoritarves concentration in a reversed biased junction - long base diode

mass action law

$$np = n_i^2$$

min concentr $n_m = \frac{n_i^2}{N_A}$ p region

current conserves through all device. current can be derived from neutral region



$$J_{TOT} = J_{diff_n}(0) + J_{diff_p}(0)$$

by Ficks law

$$J_{TOT} = +qD_n \left.\frac{dn}{dx}\right|_0 - qD_p \left.\frac{dp}{dx}\right|_0$$

$$J_{TOT} = qD_n \cdot \frac{n_0}{L_n} + qD_p \frac{p_0}{L_p}$$

in long base diodes, lifetime of carriers is

$$L_n = \sqrt{D_n \tau_n}$$ (from charge continuity eq.)

$$J_{TOT} = q \left(n_{p0} \frac{L_n}{\tau_n} + p_{n0} \frac{L_p}{\tau_p} \right)$$

$$n(0) = n_0 e^{qV_D/kT}$$ boltzmann

$$n(x) = n_0 \left(e^{qV_D/kT} - 1 \right) e^{-x/L_n} + n_0$$

when $V_D = -V_{REV}$ (reverse biased diode)

$$n(0) \simeq 0$$

$$\hookrightarrow n(x) = n_0 \left(1 - e^{-x/L_n}\right)$$

LEAKAGE REVERSE CURRENT | NO GENERATION IN DEPLETION REGION

$$J_{TOT} = \underbrace{q\left(n_{po}\frac{L_n}{\tau_n} + p_{no}\frac{L_p}{\tau_p}\right)}_{\text{neutral region contribution}} + \underbrace{q\frac{n_i}{2\tau}W_{dep}}_{\text{Generation/Recombination processes depleted region}}$$

obtain W_{dep} from poisson eq.

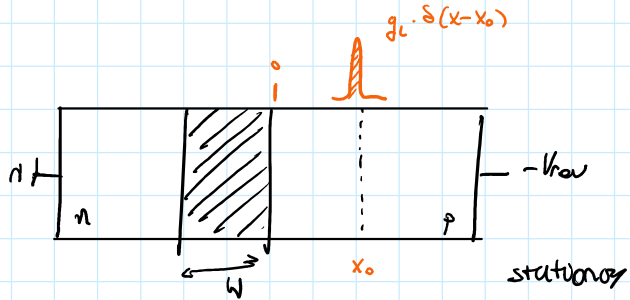
LEAKAGE REVERSE CURRENT

with a fully depleted diode, only contribution to the current are generation / recombination processes in depleted region

$$J_{leak} \simeq q\frac{n_i}{2\tau}W_{max} \quad (\omega) \qquad 1.45\ 10^{10}\,cm^{-3}$$

FULLY DEPLETED JUNCTION - REVERSE LEAKAGE CURRENT

LEAKAGE CURRENT



$$\frac{dn(x)}{dt} = \frac{1}{q}\frac{dJ}{dx} + (G-R)$$

cont eq

stationary no charge accumulation

$$0 = \frac{1}{q}\frac{d}{dx}\left(qD_n\frac{dn}{dx}\right) + G-R$$

continuity equation in p neutral region

$$\underbrace{D_n\frac{d^2n(x)}{dx^2}}_{\frac{1}{q}\frac{dJ}{dx}} - \underbrace{\frac{n(x)-n_0}{\tau_n}}_{\substack{R\\ \text{recombination}\\ \text{rate}}} = \underbrace{-g\,\delta(x-x_0)}_{G}$$

We can obtain the current generated by this δ like collection solving the continuity equation in point x=0 as before solwing the continuity equation in point $x = 0$ as before

$$J_n = q D_n \frac{dn(0)}{dx} = q D_n \frac{n_0}{L_n} - \underline{q g_L e^{-x_0/L_n}}$$

(assuming $e^{q\frac{V_{rev}}{kT}} \sim 0$

Why??

from this term

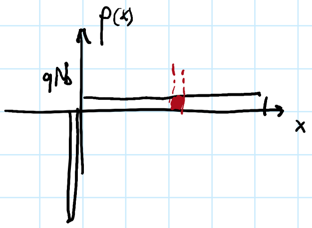
we obtain that generated current is ~ detectable when collection happens in depleted region or at $x < L_n$ in the neutral region

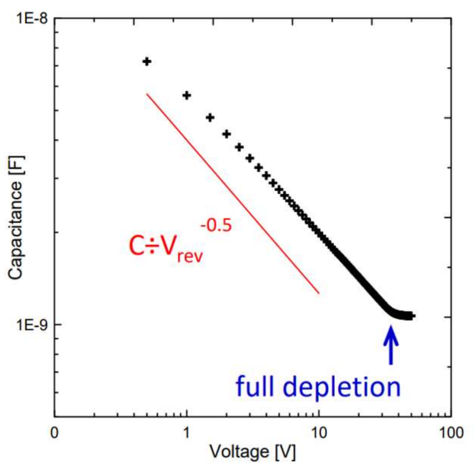
From differential equation solution

NEUTRAL REGION DETECTION

$$C_D' = \frac{dQ}{dV}\bigg|_{V=V_R} \cdot \frac{1}{A} \simeq \frac{\varepsilon_0 \varepsilon_r}{w_{dep}(V_R)} \sim \frac{\varepsilon_0 \varepsilon_r}{x_n(V_R)} \quad \alpha \sqrt{\frac{N_D}{V_R}}$$

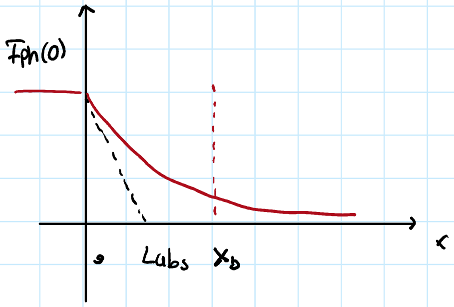
unilateral junction





Capacitance goes down with increase of w_{dep}

DEPETED REGION CAPACITANCE



$$I_{ph}(x) = I_{ph}(0)\, e^{-\mu\rho x}$$

attenuation / absoption lenght

ore

DETECTION / QUANTUM EFFICIENCY

$$\eta = \frac{I_{ph}(0) - I_{ph}(x_0)}{I_{ph}(0)}$$

$$\eta = 1 - e^{-\mu \rho x} = 1 - e^{-x_D / L_{abs}}$$

We should size

$x_D \underset{few}{\sim} L_{abs}$

=> so all particles absorbed are generating current signal

depletion region dependent on electronics of detector V_{bias} N_D	size $\Longleftarrow$	ABSORPTION LENGHT light - material dependent on μ ρ material

DEPLETED REGION SIZING

HIGH RESISTIVITY LOW DOPING DETECTORS

low doping

=> allows for fully depleted junctions without exceeding high E_{max}

?

true?

RESISTIVITY DEPENDS ON DOPING

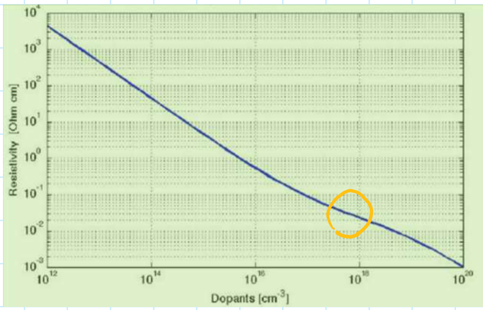
$$\rho = \frac{1}{q \boxed{\mu} \boxed{N}}$$

res

$\boxed{N}$ direct dependence on doping

$N \uparrow$ $\rho \downarrow$

more carriers



$\boxed{\mu}$ $N \uparrow$ $\mu \downarrow$ $\rho \uparrow$

mobility lowers as there are more scattering effects

2nd order effect

RESISTIVITY

---

low doping limit

↳ we cannot realistically go under 10^{13} cm^{-3}

low doping limit

↳ we cannot realistically go under 10^{12} cm-3 of doping

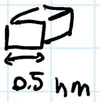
$(n_i = 1.45 \cdot 10^{10}\ \text{cm}^{-3})$

↳ in perspective $10^{12}\ \text{cm}^{-3}$ is an extremely pure silicon cristal - too pure to obtain!

$10^{12}\ \text{cm}^{-3} \rightarrow 1 \cdot \mu\text{m}^{-3}$ dopant concentration

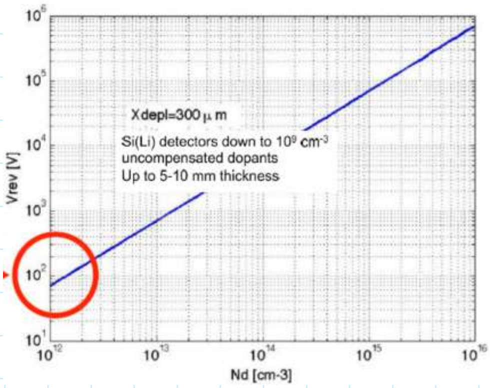
=> ~~1 dopant per μm^3~~

In $1\mu m^3$ there are



$$\Rightarrow \quad \frac{1\ \mu m^3}{(0.5)^3 \cdot nm^3} = 8 \cdot 10^9 \text{ silicon lattice units}$$

LOW DOPING LIMIT

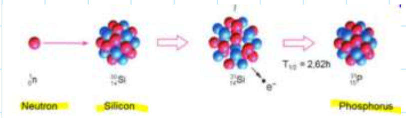


Required to keep reasonable voltage / Field values!

$$E_{max} = \sqrt{\frac{2q}{\varepsilon} N_D V_{rev}}$$

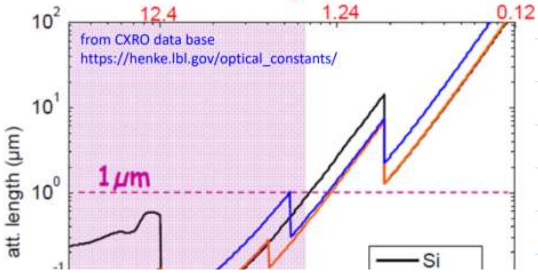
→ curiosity: high purity silicon detector grade is produced using

NTD (neutron transmutation doping)



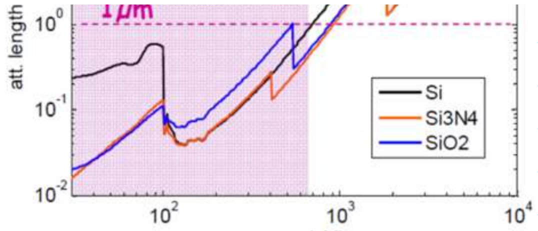
(not using implantation or ~~deposition~~ of dopants)

LOW DOPING NECESSITY



Attenuation Lenght - Energy (contrary to visible light)

Attenuation lenght lingers with energy in X RAYS

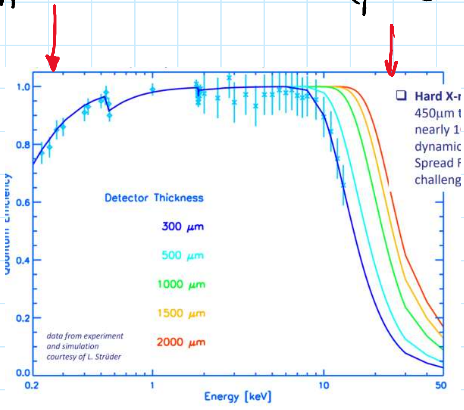


energy in X RAYS ↗

=> more energenetic photons travel further in the device

x too small Labs
photon is stopped too soon in dead layer or a low field region

x too large Labs
photon may not interact with the detector (pass through)



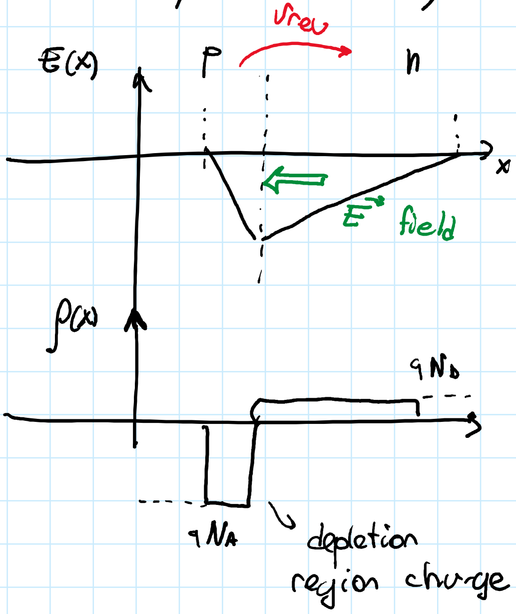
ABSORBTION LENGHT w ENERGY

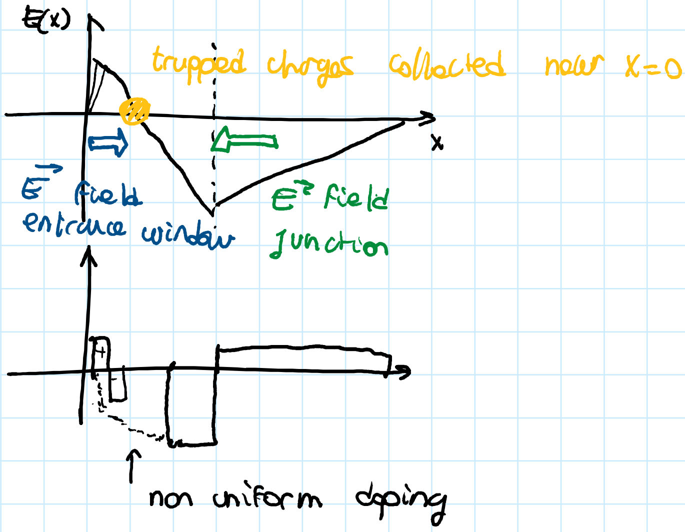
---

x high energy inefficency can be solved only through a deeper detector ⟷ technology process limit (2mm) (wafer thickness)

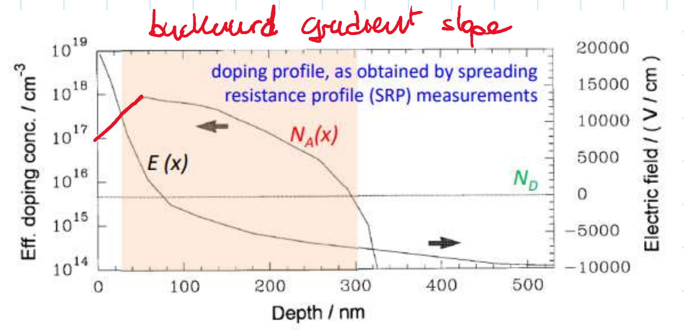
or using high z detector (higher stopping power)

x low energy inefficiency : ENTRANCE WINDOW





the small backward E field is created due to doping process - non idealities



backward gradient slope

rem. Ficks law

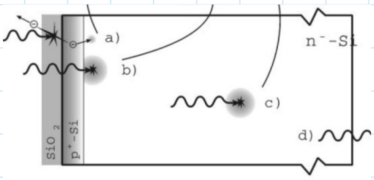
$$J_{diff} = q D_n \frac{dp}{dx}$$

$\uparrow$ non-0

=> a sort of junction is created

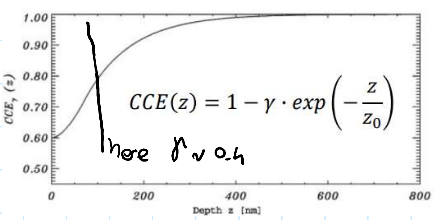
=> contrast electric field

other imperfections are:



- oxide guard on wafer

the incomplete charge collection can be modeled as empirically



here $\gamma \sim 0.4$

$\gamma = 0.09$: maximum lost charge fraction

$z_0 = 100nm$

$CCE(0) = 1 - \gamma$

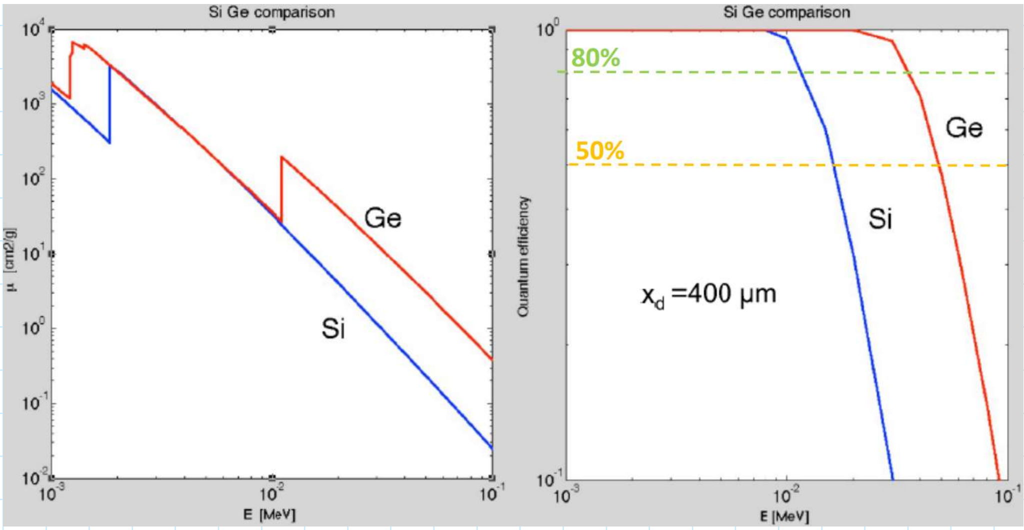
ENTRANCE WINDOW

High Z detector are useful for their higher absorption coefficient (μ)

=> they are able to capture more efficiently photons

=> same thickness (process limited) → higer detectable energy

Z: atomic number



- for example, Germanium suitable for γ-rays

Beware however for bandgap:

- lower bandgap means higher leakage current!

=> this detectors (lower bandgap) have a higher lkg current

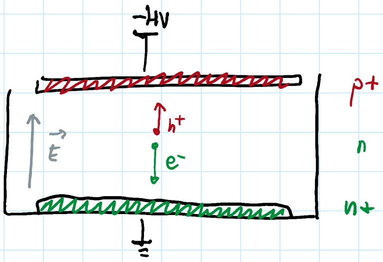
=> higher noise

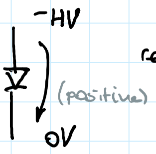
to mitigate, they must be cooled extensively (77K)

2D DETECTORS

Tuesday, May 14, 2024 11:32 AM

PIN DIODE DETECTOR





reverse biased

ANODE (electron collector)

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

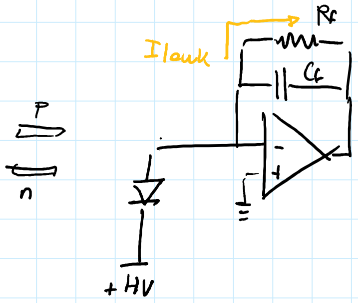
Capacitance

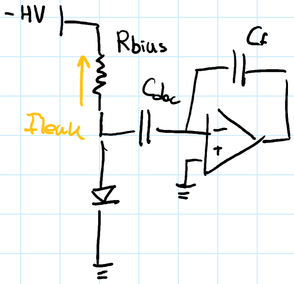
$$d_{dep} = \sqrt{\frac{2\varepsilon\varepsilon_0(N_A+N_D)}{q N_A N_D}(V_{bi}-V_{rev})} \Rightarrow C_{dep} = \frac{\varepsilon_0\varepsilon_r A}{d}$$

$\Uparrow$ depletion region width

PIN DETECTOR

DETECTOR BIASING

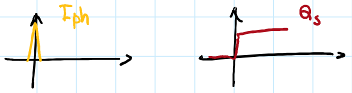




DC: leakage current

$$\Rightarrow V_{out} = I_{lk} \cdot R_f$$

AC PULSE : charge detection



$$\Rightarrow V_{out}(\overset{+}{0}) = I_{ph} \cdot \left(\frac{R_f}{1+sR_fC_f}\right) \sim I_{ph}\frac{1}{sC_f} = \frac{Q_{ph}}{C_f}$$

DC: leakage current

R_{bias} big $\Rightarrow$ tuned to I_{leak} $\longrightarrow$ $I_{LK} = -J_0 A$

$\Rightarrow$ DECOUPLED from V_{out}

AC PULSE

$$V_{out} = I_{ph} \cdot /_{sC_f} = \frac{Q_{ph}}{C_f}$$

$$V_{rev} = HV - J_0 A R_{bias}$$

AC-DC BIASING

The detector (that can be divided in strips) is biased through contact

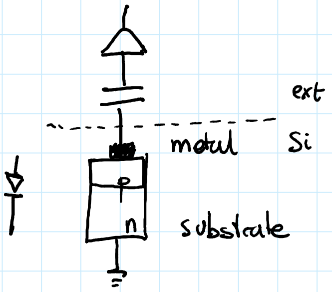
regarding AC biasing (with capacitor on diode) the biasing can be made in 2 ways

DC* COUPLING

AC COUPLING

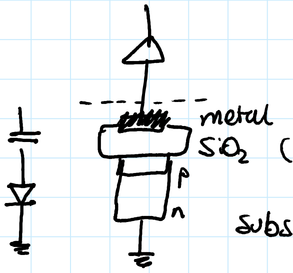
(*see later implementation)

DC* COUPLING



- external capacitor
- external bias (as seen beefore)

AC COUPLING
(*see later implementation)

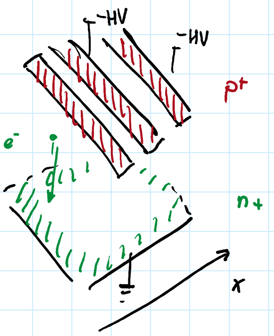


- capacitor integrated and interleaved with pn junction
- must be biased internally (diode cathode cannot be reached from ext to bias HV)

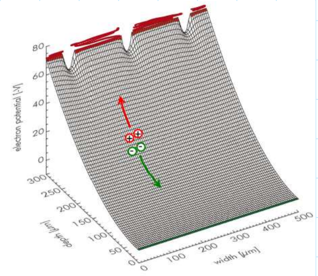
EXTERNAL - INTERNAL COUPLING

POSITION SENSING WITH STRIPS

1D DETECTOR STRIP



↑ h+ enters in one of the strip along x axis
=> 1D position detection

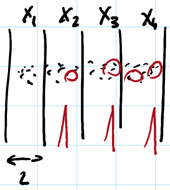


~ fully depleted diode

PIN DIODE FOR 1D POSITION DETECT.

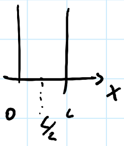
1D data can be extracted to get the position of the photon hit

1) Digital readout
when $L >> \sigma_x$ charge
(L strip >> charge cloud)



$$\boxed{\sigma_x^2 = \frac{1}{L}\int_{-L/2}^{+L/2} x^2 dx = \frac{L^2}{12}}$$

↑ quantization noise



$p(x) =$

$\uparrow$ quantization noise

/2

$p(x) =$ 

$\sigma^2 = E(x^2) - [E(x)]^2$

$\uparrow$

$\frac{1}{x}\left(\int x^2 dx\right)$

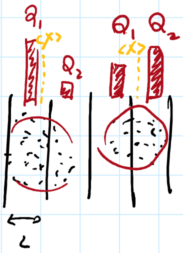
$\langle x \rangle = x_{i\,\text{center}}$

$\Uparrow$

resolution => strip width

2) Interpolation of charge cloud

$L \ll \sigma_x$ charge



$$\langle x \rangle = \frac{x_1 Q_1 + x_2 Q_2}{Q_{TOT}} = \frac{x_1 Q_1 + (x_1 + L) Q_2}{Q_{TOT}}$$

$$= x_1 + \frac{Q_2}{Q_1 + Q_2} L$$

"infinite" resolution

=> dependent on SNR we use to

read Q_1, Q_2 ..

RESOLUTION CAN BE BETTER THAN STRIP WIDTH

$$\sigma_x^2 = \frac{ENC^2}{Q^2} \cdot L^2 = \frac{L^2}{(SNR)^2}$$

- use bigger strips
- low SNR

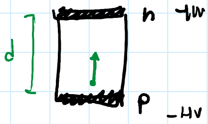
=> better result!

for cloud size, use brownian (thermal) motion

ε ??

max drift time $= \tau_n = \frac{\cancel{\varepsilon}}{\mu_n q N_D}$ $\qquad \vec{v} = \mu \vec{E}$

max drift time, in detector



$$T_{drift} = \frac{d}{\vec{v}} = \frac{d}{\mu \vec{E}} = \frac{d}{\mu q N_d d / \varepsilon} = \frac{\varepsilon}{\mu q N_D}$$

$E = \int \frac{\rho}{\varepsilon} dx$ (assuming const. doping)

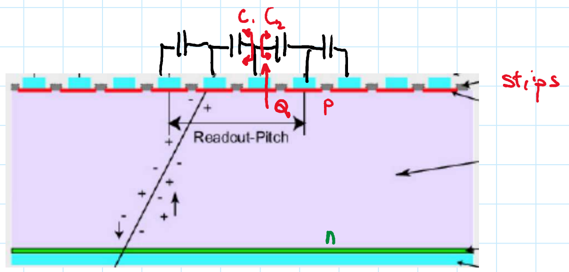
$= \rho/\varepsilon \, d = \frac{q N_d}{\varepsilon} d$

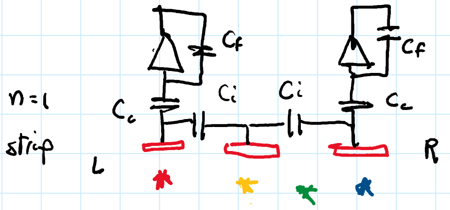
$\Delta x_{diff} = \sqrt{2 D_n T_n}$ → brownian motion gaussian FWHM

https://en.wikipedia.org/wiki/Fick%27s_laws_of_diffusion#Example_solution_2:_Brownian_particle_and_mean_squared_displacement

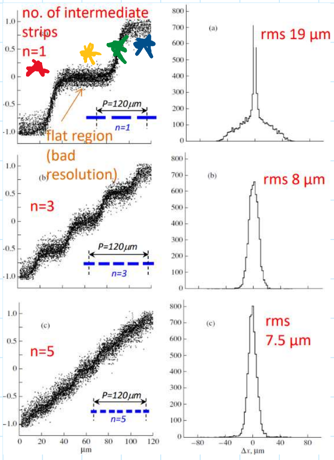
CHARGE READOUT METHODS

interpolation readout can be done using capacitances
- capacitive coupling





$$y: \frac{Q_R - Q_L}{Q_R + Q_L}$$



★ charge shared equally between R L

★ v charge all on R

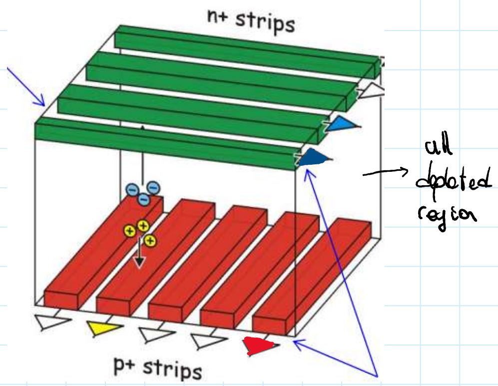
★ ~ charge all on L

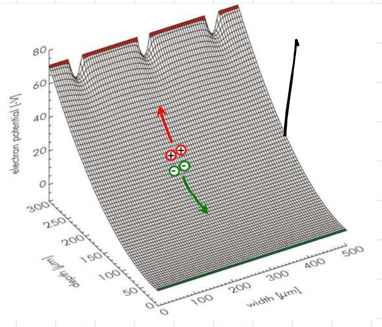
(consider as a virtual ground R, L connection post Cc approximation, as Cc big??)

★ charge sharing happens between the strips (interpolation regime

INTERPOLATION READOUT

2D DETECTOR STRIP





We can exploit having 2 different carriers to have 2D position sensing, using strips on both sides

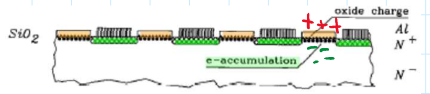
2D STRIP DET.

Introducing n+ strips causes a problem. As oxide is always deposited onto the surface (Required by process)

Introducing n+ strips causes a problem. As oxide is always deposited onto the surface (Required by process)

SiO_2 contains ⊕ trapped charges

=> INSULATION ISSUE

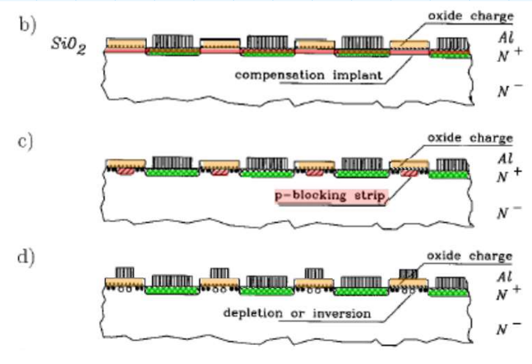


(+) trapped charges attract
(−) electron carriers

this creates two main problems

- carriers (electrons) remain trapped under strip-to-strip spaces
- the strips are no more insulated as a channel forms between strips (few ~ kΩ)
  - ↳ this may favor spread of carriers and lower position precision

Possible solutions are:



p+ spray: replace free electron (−) accumulation with N_A (⊖ fixed charge)

Same principle, but p+ is implanted and not on all surface

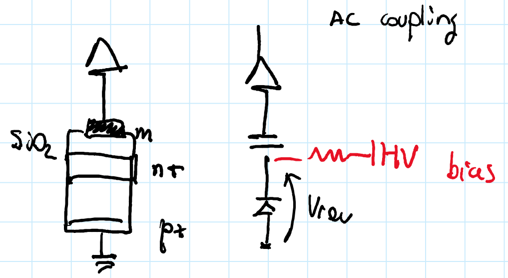
A field effect (like in mos) can be used to invert the layer, removing (−) accumulation

(on p+ strips we don't have this problem, as they are interfaced with n- substrate, so the inter-strip space is depleted - no free carrier to accumulate underneath)

SiO₂ INSULATION ISSUE

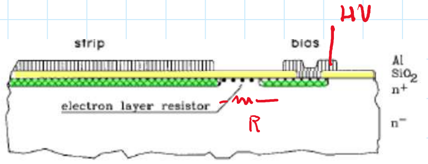
---

* AC BIASED strips

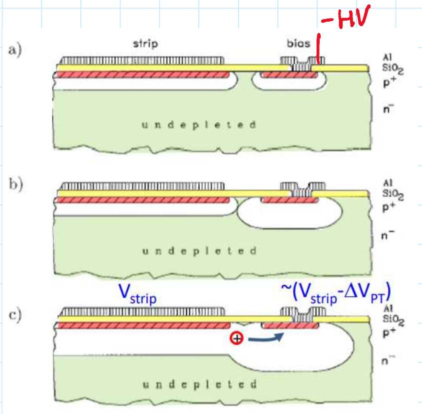


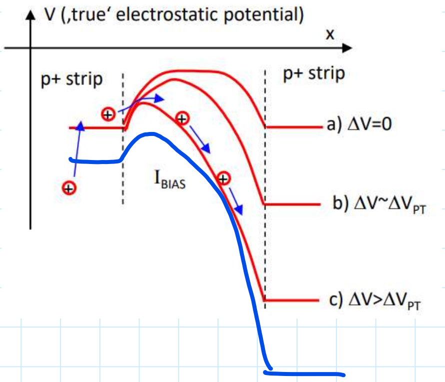
1) POLYSILICON RESISTOR : connected with n+ can brought out to a metal contact

2) ELECTRON ACCUMULATION LAYER RESISTANCE
by exploiting SiO_2 positive trapped charges. creating an accumulation (resistive layer)



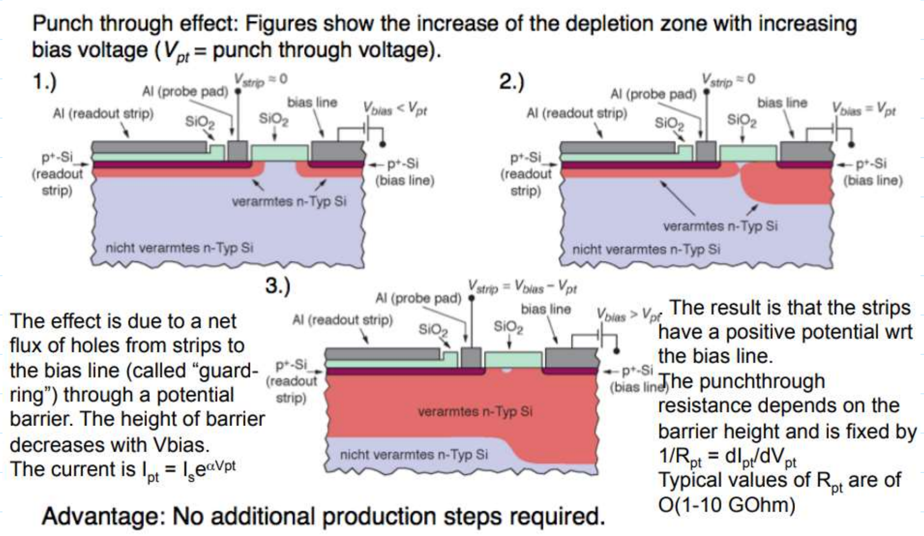
3) P+ BIASIN BY PUNCH THROUGH (on p+ strips - other side)





when punch through reached, the strips are like "shorted out" (zener diode like)

=> so pulling further the BIAS voltage drags down the strip voltage



Punch through effect: Figures show the increase of the depletion zone with increasing bias voltage (V_{pt} = punch through voltage).

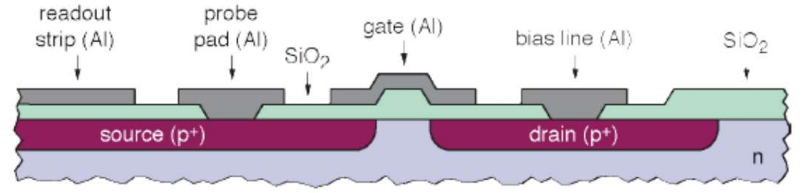
The effect is due to a net flux of holes from strips to the bias line (called "guard-ring") through a potential barrier. The height of barrier decreases with Vbias.
The current is $I_{pt} = I_s e^{\alpha V_{pt}}$

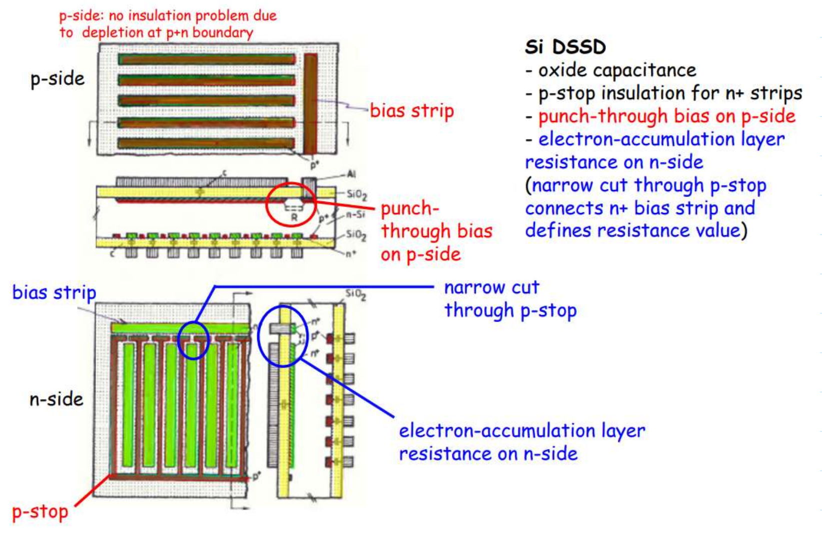
The result is that the strips have a positive potential wrt the bias line.
The punchthrough resistance depends on the barrier height and is fixed by $1/R_{pt} = dI_{pt}/dV_{pt}$
Typical values of R_{pt} are of O(1-10 GOhm)

Advantage: No additional production steps required.

4) FOXFET

Create a resistive channel using FET gate. The resistance is variable and can be modulated

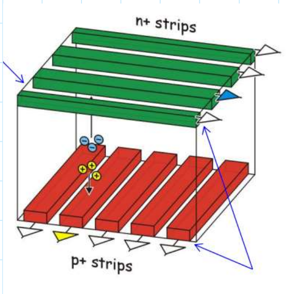




Si DSSD
- oxide capacitance
- p-stop insulation for n+ strips
- punch-through bias on p-side
- electron-accumulation layer resistance on n-side (narrow cut through p-stop connects n+ bias strip and defines resistance value)

BIASING METHODS

STRIP DETECTOR TOPOLOGIES



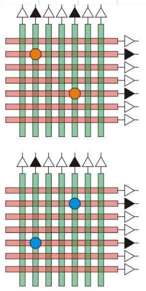
already seen

DSSD Double side Silicon Drift detector

PRO
- simple
- few readout electronics

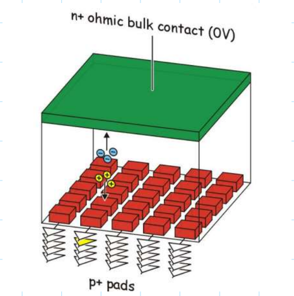
CON
- n+ strip insulation problem
- simultaneous event corruption

↳ not suitable for high occupancy measurements



DSSD

P pixel on N substrate (p on n)



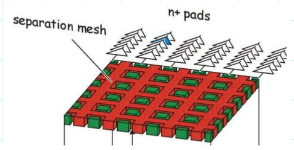
PRO
- solves n+ strip problems
- solves sim events

CON
- n^2 electronics readout instead of 2n

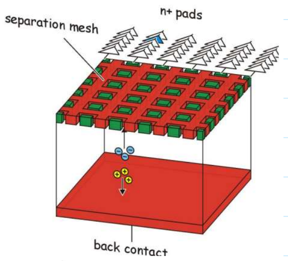
↓

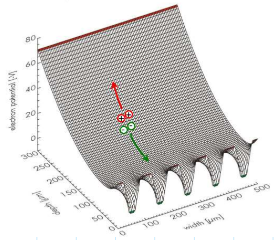
+ POWER consumption
+ interconnections

PON N



n pixels but with p+ mesh to avoid accumulation problems on n strips

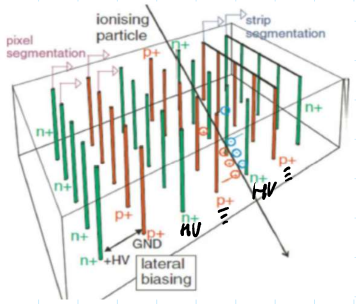




n pixels but with p+ mesh to avoid accumulation problems on n strips

same as P on N

N on N



3D Detector

PRO	con
• lower voltage need (closer strips)	• larger capacitance
• signal amplitude and charge collection can be chosen separately	

BEFORE, ON PLANAR DETECTOR

more deep => more charges generated on trepassing

more deep => more charge collection distance (more cloud ΔX, collection time) (bad!)

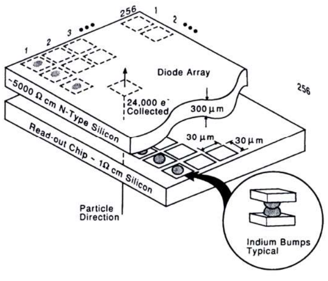
TRADE OFF

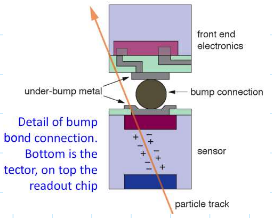
3D detector

deeper => charges more generated trepassing

charge collection distance fixed by distance between n and p trenches

3D DETECTORS





Detail of bump bond connection. Bottom is the tector, on top the readout chip

connection method for a strip detector to a readout electronics chip pro:

- possibility to choose different purity grade silicon $
- different process / technology too!

HYBRID PIXEL

ELECTRONICS

Sunday, June 30, 2024 11:40 AM

ENERGY RESOLUTION

Here we are interested in the ability to distingish different photon energies, and count the intensity of photons at each energy.

that gives the ability to do spectroscopy, so to visualize the spectrum of the incoming light.

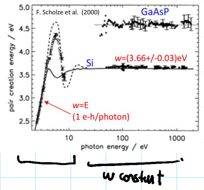
Spectrum example:

1) good energy resolution
2) bad energy resolution

ENERGY SPECTRUM

Energy of an impinging photon can be derived from the number of e-h pairs generated inside the detector.

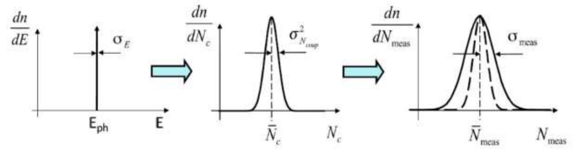
=> see PART I : mean creation energy



$$\langle N_{e\text{-}h} \rangle = \frac{E}{\langle W(E) \rangle} = \frac{h\nu}{\langle W(E) \rangle}$$

$$\sigma_{N_{eh}}^2 = F \cdot \langle N_{eh} \rangle$$

dn/dE spectrum



n photons

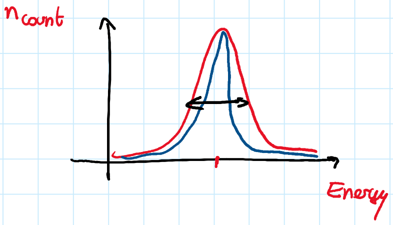
n photons on energy

n photons on N_{eh} generated (at energy E)

n photons on N_{eh} read by electronics

$\sigma_E^2 = 0$ $+ \sigma_{Noise}^2$ FANO $+ \sigma_{Noise}^2$ electronics

ENERGY RESOLUTION



$$FWHM_E = 2.335 \cdot W \, \sigma_{N_{eh}}$$

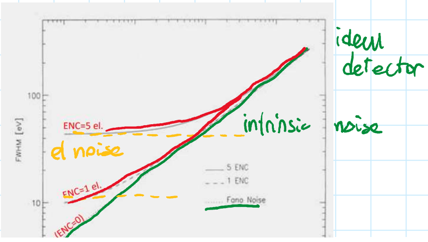
W ↑ mean creat Energy

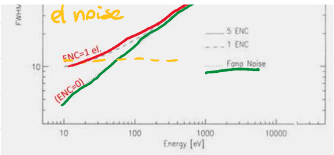
$\sigma_{N_{eh}}$ ↑ generated carriers

$$= 2.335 \cdot W \sqrt{\frac{F \cdot E}{w} + \left(\frac{ENC}{q}\right)^2}$$

↑ Fano (intrinsic) contribution

↑ electronics contributions





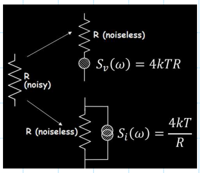
(intrinsic) contribution   contributions

$$\sigma_n^2 = \frac{F \cdot E}{\omega} + \left(\frac{ENC}{q}\right)^2$$

FANO - ELECTRONICS NOISE CONTRIB.

Noise in electronics components.
Note that the given power spectral densities are UNILATERAL

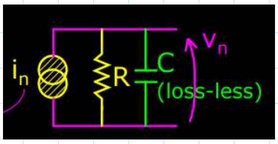
RESISTOR



$$S_{v_u}(f) = 4kTR$$

$$S_{i_u}(f) = \frac{4kT}{R}$$

CAPACITOR (LOSSY)



shunt resistance across capacitance - due to dielectric relaxation

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \qquad \tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

admittance
↓

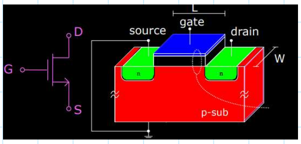
$$Y = j\omega(\varepsilon_r' - j\varepsilon_r'')\frac{\varepsilon_0 A}{d} = \underbrace{j\omega C}_{\text{comlex}} + \underbrace{\omega C \tan(\delta)}_{\text{reall}}$$

$$S_i(\omega) = \frac{4kT}{R} = 4kT \cdot \omega C \tan(\delta)$$

$$S_v(\omega) = 4kT\frac{\cancel{\tan(\delta)}}{\omega C}\tan(\delta) \; ?$$

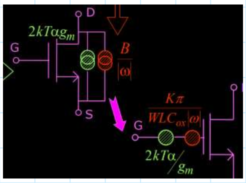
=> lossy capacitors introduce 1/f noise

MOSFET



They introduce white and 1/f noise.

1/f contribution is often significant and must be addressed

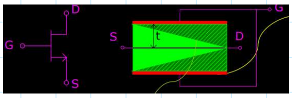


$$S_{w_0}(f) = 4kT\alpha g_m \quad (A^2/Hz)$$

sat-region $\alpha \sim \frac{2}{3} \propto 2$

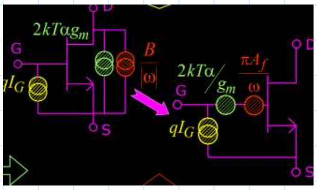
$$S_{w_0}(f) = 4kT/R_{chn} \quad (A^2/Hz)$$

JFET



Junction FET exploit a pn junction depletion region to close the channel between source and drain.
this avoid using a gate capacitance, introducing oxide and trapped charges giving rise to 1/f noise

However pn junctions add shot noise, due to leakage current between gate and S/D



$$S_{w_0} = 4kT\gamma g_m \quad (A^2/Hz)$$

$S_{w_0}(f) = 4KT/R_{cha}$ (A^2/Hz)

ohmic region

ω

$S_I = \frac{S_w \cdot W_c}{\omega}$ (A^2/Hz)

↓ (doing $\frac{S_I}{g_m^2}$ + physics on Gate)

$S_V : \frac{K}{C_{ox}' WL} \cdot \frac{1}{\omega}$ (V^2/Hz)

1/f

$S_{w_0} = 4KT\gamma g_m$ (A^2/Hz)

ω

$S_{1/f_c} \cdot \frac{S_w \cdot W_c}{\omega}$ (A^2/Hz)

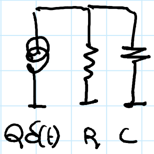
1/f

$S_{s_I} = 2qI$ (A^2/Hz)

shot

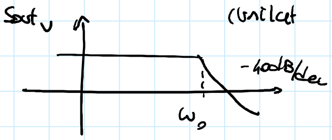
beware btwn gate / drain

NOISE IN DEVICES



$$S_V = S_i \cdot \left(\frac{R}{1+sRC}\right)^2 = \frac{4KT}{R} \cdot \frac{R^2}{(1+\omega^2R^2C^2)}$$

$$= \frac{4KTR}{(1+\omega^2R^2C^2)}$$



$$\sigma_V^2 : \int_{-\infty}^{+\infty} S_{out}(f)\,df = \overset{\text{equ}}{\text{noise BW}} : 4KTR \cdot \underbrace{\frac{\pi}{2} \cdot \frac{1}{2\pi RC}}_{1/4RC} = \frac{KT}{C}$$

noise charge on capacitor

$C = \frac{Q}{\Delta V}$

$$\sigma_c^2 = (C^2) \cdot \sigma_V^2 = KTC$$

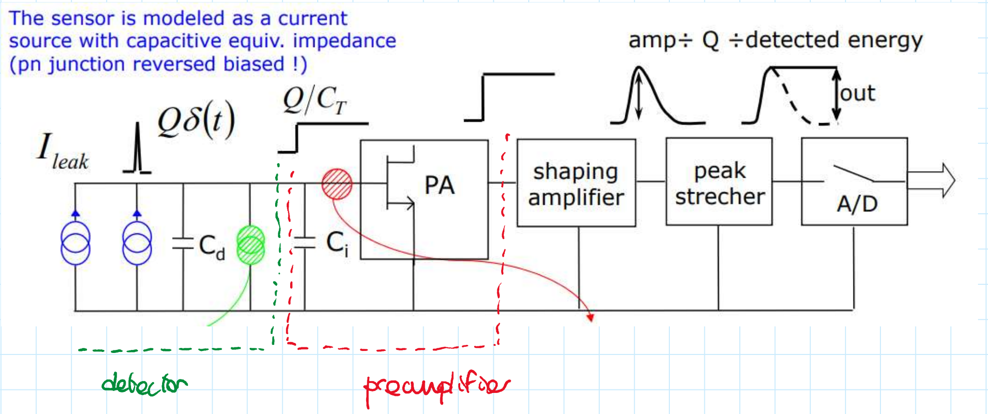
independent on R

because proportionally increase noise and inverse proportionaly decrease BW, does not impact on noise MSV

KTC RC FILTER NOISE

NOISE PERFORMANCE

MEASUREMENT SYSTEM TOPOLOGY



The sensor is modeled as a current source with capacitive equiv. impedance (pn junction reversed biased !)

amp÷ Q ÷detected energy

signal:
can be modeled
as a pulse like
current delivering

typically a
common source
FET

can be modelled
as a pulse like current delivering a fixed amount charge

- I_{leak} : junction leakage
- C_d junction depletion region capacitance
- noise : shot noise from leakage current

common source FET

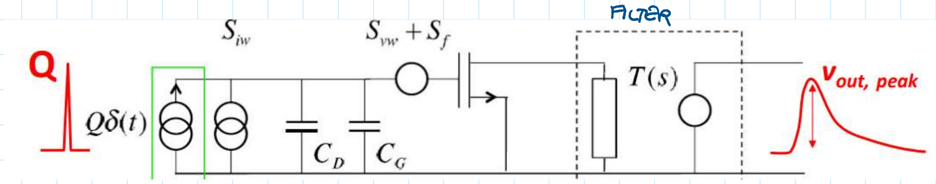
- C_I gate capac.
- series Voltage ↓ noise

$$\frac{2KT\alpha}{g_m} + \frac{1}{2}\frac{A_f}{f} \left[V^2/f \right]$$

(bilateral)

ELECTRONICS OVERVIEW

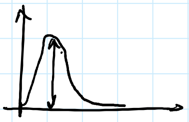
pay attention, in following calculations we use bilateral spectral densities



$F(S) = Q$ Ampere
(delta => constant in f)
(step => integral of δ
=> 1/s · constant)

$$I_{drain} = Q \cdot \frac{1}{s(C_D+C_G)} \cdot g_m$$

$$V_{out}(s) = Q \cdot \frac{1}{s(C_g+C_G)} \cdot g_m \, T(s) = Q \cdot \underbrace{H(s)}$$



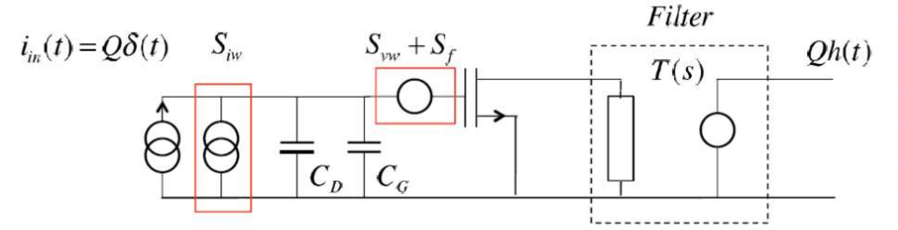
$V_{out\ max}$ α Q

$$V_{out}(t) = \frac{Q}{(C_D+C_G)} g_m \cdot \mathcal{L}^{-1}\left(\frac{1}{s}T(s)\right)$$

↑ filter

SIGNAL OUTPUT

using bilateral spectral densities : unilateral /2

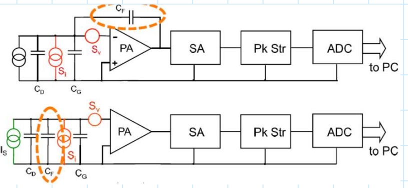


S_{iw} : detector shot noise = $q I_{leak}$ b

S_{vw} : input refered FET white noise = $\frac{2KT\alpha}{g_m}$ a

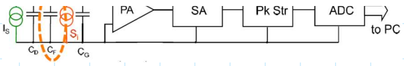
S_f : input refered FET 1/f noise = $\frac{1}{2}\frac{A_f}{f} = \frac{\pi A_f}{\omega}$ c

noise power spectral density



↱ charge amplifier

i noise power spectral density



C_D = Cstray + Cdetector + Cfeedback ↱ charge amplifier feedback of ANY why ⟶ FOR THE SNR calculation (NOT SIGNAL OR NOISE ONLY SNR) C_F can be considered parallel to detector

We can then compute the power spectral density at the output

$$S_{Vout}(\omega) = \left(\frac{2kT\alpha}{g_m} + \frac{\pi A_f}{\omega} + qI_k \cdot \frac{1}{\omega^2(C_G+C_D)^2}\right) g_m^2 \, |T(j\omega)|^2$$

to get the noise power (variance of the noise)

$$\sigma^2_{n\,out} = \int_{-\infty}^{+\infty} S_{Vout}\, df = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{Vout}\, d\omega \qquad d\omega = d(2\pi f)$$

To quickly evaluate signal impact it is common to input refer the noise sources / power spectral densities

=> input reffered current (equal to signal)

$$S_{V_{out}} = \underbrace{\left|\left(\frac{2kT\alpha}{g_m} + \frac{\pi A_f}{\omega}\right)\cdot\omega^2(C_D+C_G)^2 + qI_k\right|}_{\text{input refered power spectral densities}} \quad |H(s)|^2$$

NOISE OUTPUT

Signal to noise ratio is computed

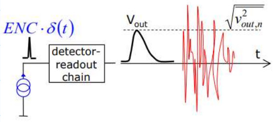
$$\left(\frac{S}{N}\right)^2 = \frac{Q^2\,\dfrac{g_m^2}{(C_D+C_G)^2}\cdot \max\left(\mathcal{L}^{-1}\left[T(s)/s\right]\right)}{\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{Vout}(\omega)\,d\omega} \qquad \begin{matrix} \sim 1 \quad V_{out}^2 \text{(peak)} \\ \\ \sigma^2_{out} \end{matrix}$$

$$\left(\frac{S}{N}\right)^2 = \frac{Q^2\,\dfrac{g_m^2}{(C_D+C_G)^2}}{\frac{1}{2\pi}\int S_{Vout}(\omega)\,d\omega}$$

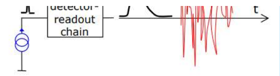
We can define the EQUIVALENT NOISE CHARGE figure of merit

if we impose a $S/N = 1$ we can find the minimum detectable signal at the input finding the needed signal charge to equal the noise amplitude

$$1 = \frac{(ENC)^2\cdot\dfrac{g_m^2}{(C_D+C_G)^2}}{\frac{1}{2\pi}\int S_{Vout}(\omega)\,d\omega}$$



$$\frac{1}{2\pi}\int S_{vout}(\omega)\,d\omega$$



$$ENC^2 = (C_D+C_G)^2\frac{2KT\alpha}{g_m}\cdot\frac{1}{2\pi}\int_{-\infty}^{+\infty}|T(j\omega)|^2 d\omega + (C_G+C_D)^2\cdot\pi A_f\frac{1}{2\pi}\int\frac{1}{\omega}|T(j\omega)|^2 d\omega$$

series white (voltage) noise — series 1/f voltage noise

$$+\quad qI\cdot\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{\omega^2}|T(j\omega)|^2 d\omega$$

parallel white (current) noise

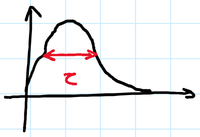
SNR / ENC CALCULATION

ENC contains different terms, each differently dependent on $T(j\omega)$ (shaping circuit)

The output pulse will be a combination of a

HPF action (to cut LF noise, such 1/f)
LPF ation (to cut WIDEBAND noise, like white noise)

The output pulse will be something like



if we want to enhance the time (τ) dependence, we can rewrite

$$\omega = \frac{x}{\tau} \qquad x = \omega\tau \qquad \frac{dx}{\tau} = d\omega$$

$$ENC^2 = (C_D+C_G)^2\frac{2KT\alpha}{g_m}\cdot\frac{1}{2\pi}\frac{1}{\tau}\int_{-\infty}^{+\infty}|T(jx)|^2 dx + (C_G+C_D)^2\cdot\pi A_f\frac{1}{2\pi}\int\frac{\cancel{\tau}}{x}|T(jx)|^2\frac{dx}{\cancel{\tau}}$$

series white (voltage) noise — series 1/f voltage noise

$$+\quad qI\cdot\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\tau^{\cancel{2}}}{x^2}|T(jx)|^2\frac{dx}{\cancel{\tau}}$$

parallel white (current) noise

$$ENC^2 = (C_D+C_G)^2 a\frac{1}{\tau}A_1 + (C_D+C_G)^2 c A_2 + b\tau A_3$$

series white noise — series 1/f noise — parallel white noise

note the dependence on τ = shaping time

$\uparrow\tau$ $\downarrow$ series (voltage) white

$\times\tau$ 1/f noise

$\times Z$ $\quad\quad$ $1/f$ noise

$\downarrow Z$ $\quad\quad$ $\downarrow$ parallel white noise

while the $A_1\, A_2\, A_3$ term are dependent only on the shaper output "shape", not on the absolute time of the shape

SHAPER IMPACT

---

the filter has so an impact on SNR, as its filtering action must enhance SNR

Lets consider only white noise at first

$$SNR = \frac{Q^2}{\underbrace{\frac{2kT\alpha}{g_m} C_T^2}_{\text{constant terms out integral}} \cdot \int_{-\infty}^{+\infty} \omega^2 |h(t)|^2 dt + qF \cdot \int_{-\infty}^{+\infty} |h(t)|^2 dt}$$

a $\quad$ b

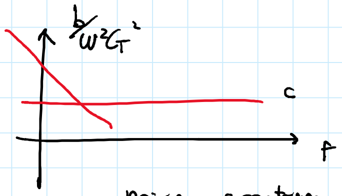
↑ derives from $K_{ww}(0) \Rightarrow \int h(t)^2 dt$

- - - - - - - series white $\quad$ - - - - - - - parallel white

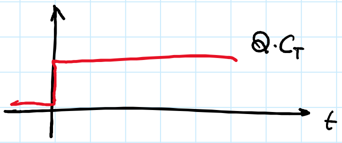
NOISE Power spectral density

$\downarrow$ $\omega_z = 1/\tau_c =$ $\quad$ $\tau_c = C_T \sqrt{\frac{a}{b}}$ noise corner time constant

after capacitance $\cdot \frac{1}{\omega^2 C_T^2}$

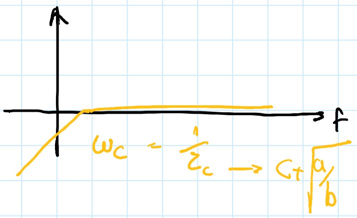


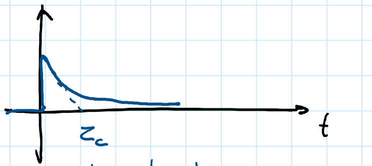
noise spectrum



signal time shape impulse to step (integration on capacitance)

following optimum filter theory, whiten noise

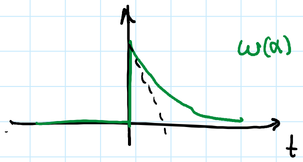




signal shape after whitening filter

and boid Filter on signal shape

=>



w(t) optimum filter shape

$$w(t) = e^{-t/\tau_c}$$

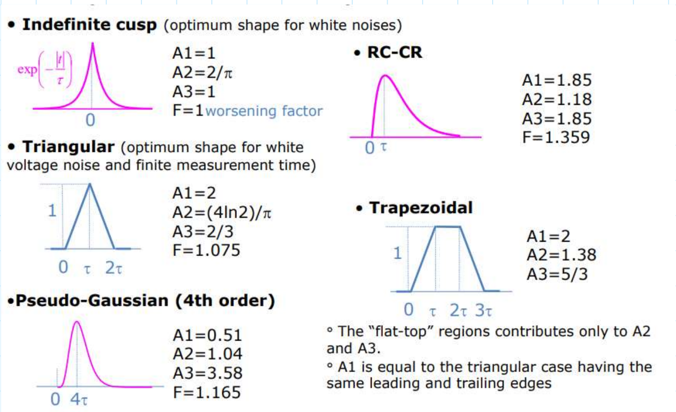
τ_c cutoff point

$$= C_T\sqrt{\frac{a}{b}}$$

OPTIMUM SHAPING ON WHITE NOISE

If 1/F noise is present and not negligible, unfortunately we cant whiten it and cant build an optimum filter

We can size and shape the filter optimizing the output



- **Indefinite cusp** (optimum shape for white noises)

A1=1
A2=2/π
A3=1
F=1 worsening factor

- **RC-CR**

A1=1.85
A2=1.18
A3=1.85
F=1.359

- **Triangular** (optimum shape for white voltage noise and finite measurement time)

A1=2
A2=(4ln2)/π
A3=2/3
F=1.075

- **Trapezoidal**

A1=2
A2=1.38
A3=5/3

◦ The "flat-top" regions contributes only to A2 and A3.
◦ A1 is equal to the triangular case having the same leading and trailing edges

- **Pseudo-Gaussian (4th order)**

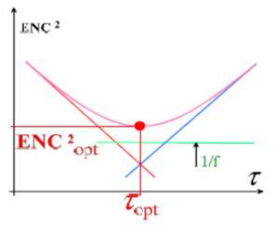
A1=0.51
A2=1.04
A3=3.58
F=1.165

OTHER FILTERS SHAPES

The shaping time can be optimized following the noise contributions. (when we cant follow the optimum filter)

$$ENC^2 = (C_D+C_G)^2 a \frac{1}{\tau} A_1 + (C_D+C_G)^2 c A_2 + b\tau A_3$$

series white series 1/f parallel white



τ_{opt} => $ENC^2_{\text{series white}} = ENC^2_{\text{par white}}$

$$\tau_{opt} = \sqrt{\frac{A_1}{A_3}}\sqrt{\frac{a}{b}}\,(C_D+C_G)$$

$$ENC^2_{opt} = c(C_D+C_G)^2 A_2 + 2\cdot\sqrt{A_1A_3}\sqrt{ab}\,(C_D+C_G)^2$$

OPTIMAL SHAPING TIME W 1/F NOISE

Input transistor parameters (preamplifier) influence the SNR via

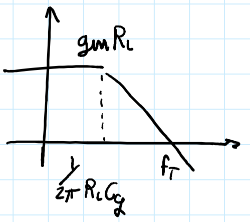
$$ENC^2 = (\boxed{C_G}+C_D)^2\,\boxed{a}\,\frac{1}{\tau}A_1 + (C_D+\boxed{C_G})^2\,\boxed{c}\,A_2 + b\tau A_3$$

$$ENC^2 = (\boxed{C_G} + C_D)^2 \boxed{a} \frac{1}{2} A_1 + (C_D + \boxed{C_G})^2 \boxed{c} A_2 + b \tau A_3$$

$$a = S_{v_{b\,white}} = \frac{2kT\alpha}{g_m} \qquad c = \pi A_f \longleftrightarrow S_{v,1/f\,bj} = \frac{\pi A_f}{\omega}$$

noise contributions

$$C_G = C'_{ox} \frac{WL}{t_{ox}}$$



Introduction (unm.edu)

MOS common source Transfer function

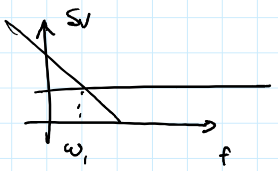
$$f_T = g_m R_L \cdot \frac{1}{2\pi R_L C_g}$$

$$\omega_T = g_m / C_g \qquad g_m = \omega_T C_g$$

gate capacitance

↑ cutoff frequency

The transistor noise is described as



to find noise corner frequency

$$\frac{\pi A_f}{\omega_c} = \frac{2kT\alpha}{g_m} \qquad \pi A_f = \frac{2kT\alpha}{\omega_T C_G}\omega_c$$

↑ 1/f noise ↑ white

$$ENC^2 = \underbrace{(C_G + C_D)^2 \cdot \frac{2kT\alpha}{\omega_T C_G}}_{\text{series white}} \cdot \frac{1}{2} A_1 + \underbrace{(C_G + C_D)^2 \cdot \pi A_f}_{\text{series } 1/f} A_2$$

$$+ \underbrace{q I_{leak} \tau A_3}_{\text{parallel white}}$$

$$ENC^2 = (C_G + C_D)^2 \frac{2kT\alpha}{\omega_T C_G} \cdot \frac{1}{2} A_1 + (C_G + C_D)^2 \frac{2kT\alpha}{\omega_T C_G} \omega_c + q I_{lk} \tau A_3$$

$$= \frac{(C_G + C_D)^2}{C_G} \frac{2kT\alpha}{\omega_T} \left(\frac{1}{2} A_1 + \omega_c A_2 \right) + q I_{lk} \tau A_3$$

assuming ω_T constant ~ bandwidth

requires fixed current density

to minimize the ENC of the transistor, we must minimize

$$\frac{(C_G + C_D)^2}{C_G} = C_D \left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}} \right) \longrightarrow \text{minimize when matching}$$

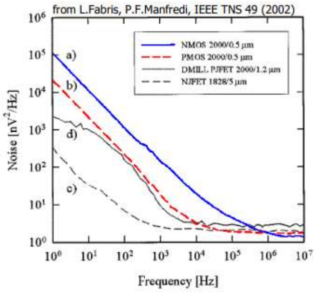
$$\frac{(C_G + C_D)}{C_G} = C_D\left(\sqrt{\frac{C_D}{C_G}} + \sqrt{\frac{C_G}{C_D}}\right) \rightarrow$$ minimize when matching

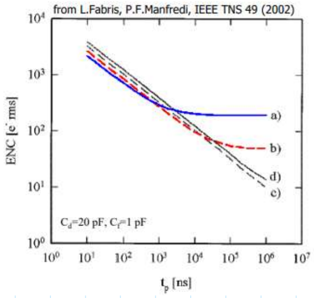
$C_G = C_D$

matching gate capacitance with detector capacitance

Keep in mind that also gate leakage current must be considered if the transistor type is JFET or maybe BJT

shot contribution $= q\left(I_{\underset{\text{detector}}{lu}} + I_{gate} + \ldots\right) 2$





MOSFET:
- higher g_m
- smaller white noise

PMOS
- lower $1/f$ noise than NMOS

if shaping times $\sim 1\mu s$

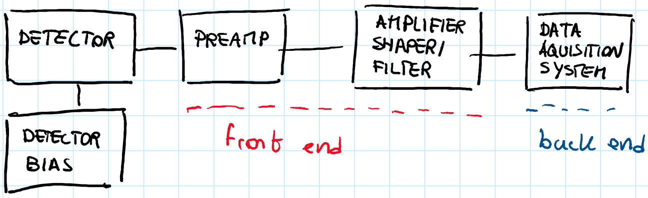
PMOS > NMOS, JFET

PREAMPLIFIER TRANSISTOR NOISE CONTRIBUTION

capacitor matching for same power dissipation ?

DO!

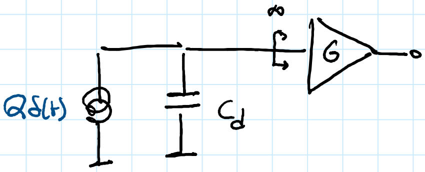
PREAMPLIFIER CIRCUIT



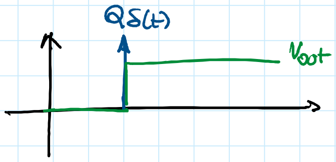
- why the preamplifier
  - → mimize the noise of the following stages via amplification
  - → minimze noise contribution of transission by placing it as close as possible to the detector

→ minimize noise contribution of transmission by placing it as close as possible to the detector
→ transmission signal driving

PREAMPLIFIER



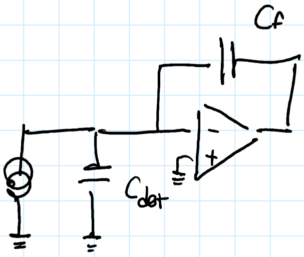
$$V_{out} = \frac{Q}{C_{det} + C_{par}} G \cdot$$



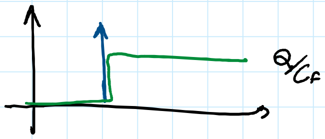
the major drawback of the voltage preamplifier is the direct dependence on

- C detector : not stable and varying with bias
- C parasitic : not stable and hardly predictable

VOLTAGE PREAMPLIFIER



$$V_{out}\Big|_{id} = \frac{Q}{C_f}$$



the charge amplifier (invented by Emilio Gatti!) solves the issue of dependence on unstable capacitance

the feedback capacitance can be freely designed and produced

CHARGE AMPLIFIER (IDEAL)

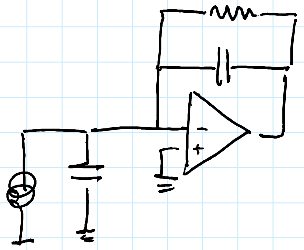
$$V_{out}\Big|_{real} = V_{out\,id} \cdot \frac{1}{1 - 1/G_{loop}} = \frac{Q}{C_f} \cdot \frac{1}{1 + \frac{1}{A \cdot \left(\frac{1/C_{in}}{1/C_{in} + 1/C_f} \right)}}$$

↑ negative G_{loop}

$$= Q/C_f \cdot \frac{1}{1 + \frac{1}{A} \cdot \frac{C_f + C_{in}}{C_f}} = \frac{Q}{C_f} \cdot \frac{1}{1 + \frac{1}{A}\left(1 + \frac{C_{det} + C_{par}}{C_f}\right)}$$

if $A >> 1 \Rightarrow V_{out}\Big|_{real} \cong V_{out}\Big|_{id} = \frac{Q}{C_f}$

FINITE GAIN FEEDBACK

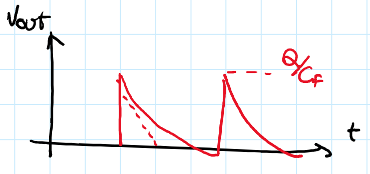


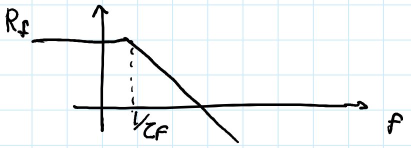
capacitor (integrator) must be reset between pulses,

otherwise multiple pulses overlap

can do it with resistance or (open mosfet in short channel technologies)

-> continuous reset





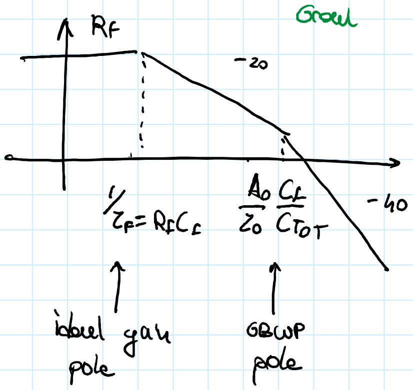
$\tau = R_f C_f$ typically $\tau \sim> 40\,\mu s$

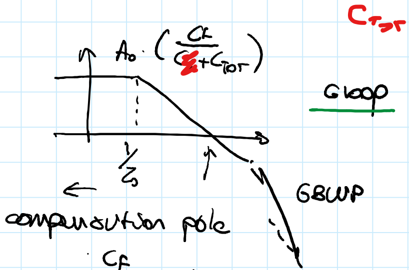
We also must consider that the amplifier often has at least one pole

$$A(s) = A_0 \cdot \frac{1}{1+s\tau_0}$$

$$\tau_0 = r_0 C_0$$

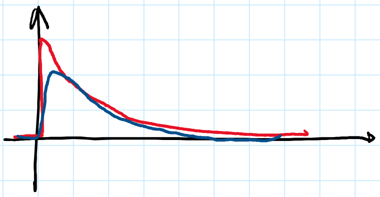
$$C_{TOT} = C_0 + C_F$$





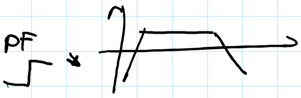
$$\simeq A_0 \frac{C_F}{C_{TOT}} \cdot \frac{1}{\tau_0} = \omega_{GBWP}$$

REAL TRANSFER FUNCTION



(finite rise time ~ < 100 ns <> GBWP)

(Also think of this time domain response as

STEP response of a LPF + HPF 

δ is derivative of step so

STEP is integral of δ

=> same time response

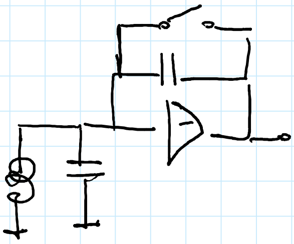
derivative (·s) 

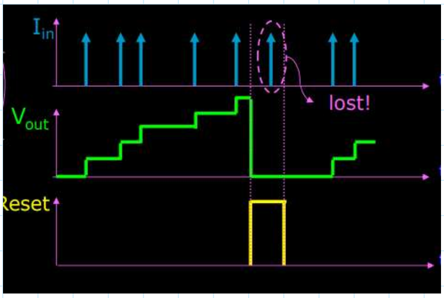
integral ·(·1/s)

REAL PREAMP TIME RESPONSE

The reset can also be pulsed /non continuos

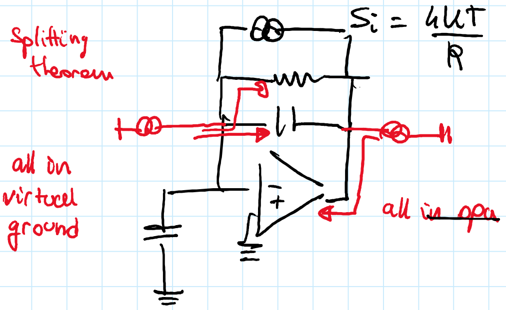
-> the capacitance can be discharged with a MOS





GATED RESET

The introduction of the reset resistor also adds a reset noise contribution



$S_i = \frac{4kT}{R}$ (unilateral)

If virtual ground ideal

$$\sigma^2_{V_{out}} = \int_0^{+\infty} S_i(f) \cdot \left| \frac{R_f}{1+sR_fC_f} \right|^2 df$$

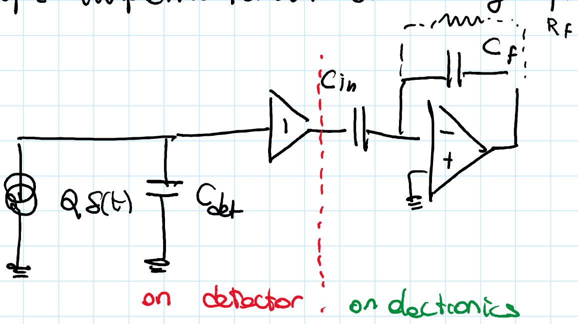
$$= \frac{4kT}{R_f} \cdot R_f^2 \cdot \underbrace{\frac{\pi}{2} \cdot \frac{1}{2\pi R_f C_f}}_{\text{Noise Eq BW}}$$

$$= \frac{kT}{C_f}$$

$$\sigma^2_{ch} = ENC^2 = \frac{kT}{C_f} \cdot C_f^2 = kTC_f$$

KTC ON RESET RESISTOR

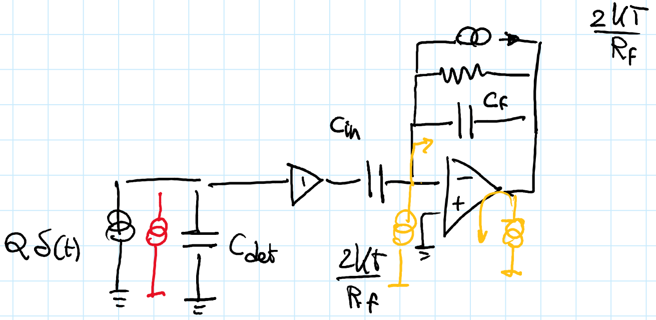
Example implementation of voltage preamplifier



$$V_{out} = \frac{Q}{C_{det}} \cdot \frac{C_{in}}{C_f}$$ assuming ideal

↳ $\left(\frac{Z_2}{Z_1}\right.$ inverting opa$\left.\right)$

NOISE contribution



input referring the noise

$$I_o = I_{in} \cdot \frac{1}{sC_{det}} \cdot \frac{1}{1/sC_{in}} = I_{in} \cdot \frac{C_{in}}{C_{det}}$$

noise

$$\star \quad S_{I_n} = \frac{2KT}{R_f} \cdot \left(\frac{C_{det}}{C_{in}}\right)^2$$

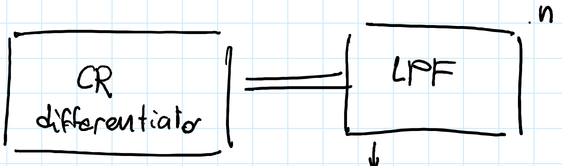
VOLTAGE PREAMP EXAMPLE

SHAPING AMPLIFIER

from the preamp, the signal comes out as a step: as the delta-pulse signal is integrated (on the detector capacitance or by a charge amplifier)

The shaper transform this step-like function in a readable pulse / signal leveraging filters to improve SNR

Typically



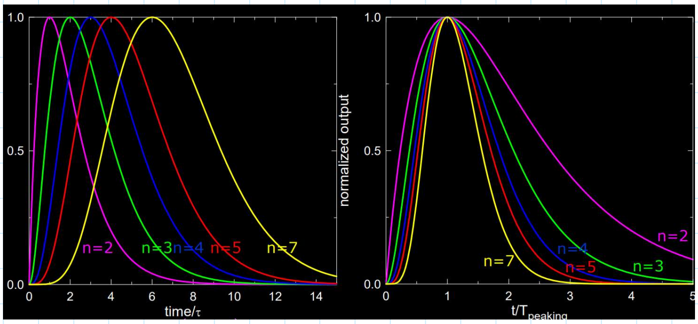
low HPF cutoff to limit 1/f, baseline components

n order LPF filter, steep as needed

$$T(s) = \frac{s\tau}{(1+s\tau)^n} \longrightarrow \text{real coincident poles}$$

other LPF transfer functions can be obtained using non real poles

(chebicev - resonant filter - bessel etc)



A high pass alternative that is a NON CONSTANT PARAMETER SIGNAL ← needs a sync signal when signal incoming

is the baseline restorer

=> cuts low frequency noise:

- 1/f
- baseline offset

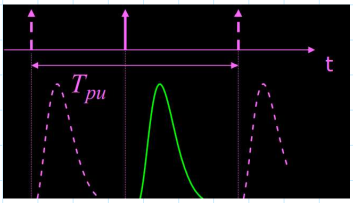
SW closed when NO SIGNAL
↳ HPF on baseline

SW open when SIGNAL
↳ no action on signal

SHAPING AMP TYPES/FILTERS

---

the shaping aupifier also has a great impact on multiple pulse detection

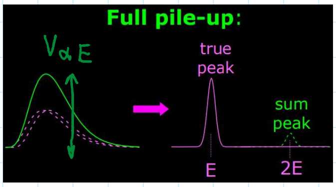


$P_{non\,pileup} = e^{-h_{in} T_{pu}}$

$P_{pileup} = 1 - P_{np}$

The pileups can be of two types:

FULL PILEUP



Two peaks almost perfectly aling giving rise to

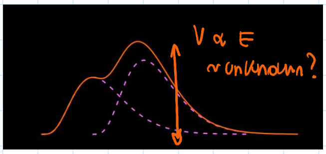
Detection of energy $E_{pu} = E_1 + E_2 = (2E)$ in this case

They can be reeognigezd for 2 reason:

- A known frequency (a combination of sum of true freqencies)
- The integral higher with a higher counting rate

- A known frequency (a combination of sum of true frequencies)
- The intensity highers with a higher counting rate (as pileups become more probable)

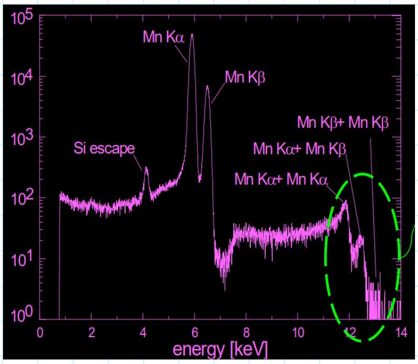
INTERMEDIATE PILEUP

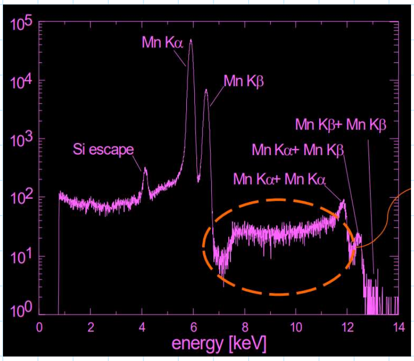


Most undesiderable becuse:

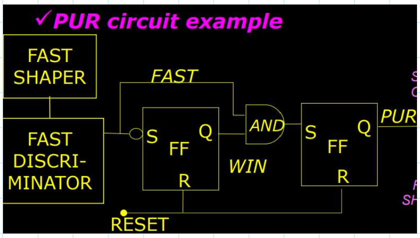
x unknown result energy (sum can be off phase by an unknown amount) ↓

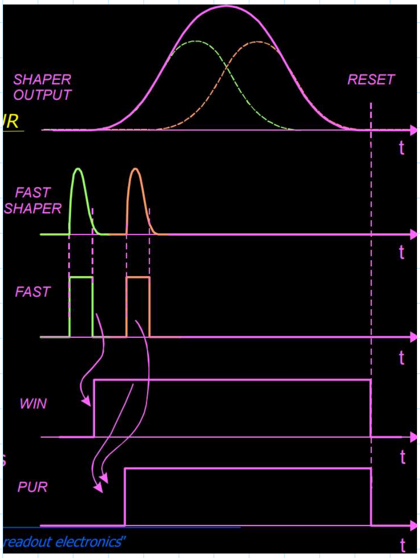
=> any amplitude between peak energy and the sum can be obtained





PILEUP





A Pile Up Rejector circuit can be used to detect pileups and take action

=> Fast shaper allows the detection of pileps in the "normal" shaper time

↳ the fast shaper output must be compared with a threshold threshold must be chosen carefully as a fast shaper output is more noisy

- too low: false positive rejections
- too high: pileups may not trigger the PUR

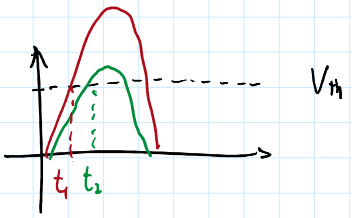
PILEUP REJECTOR

TIME MEASUREMENTS

TIME MEASUREMENTS

If we are interested in measuring the time of arrival of a photon, we must add a time detector circuit.

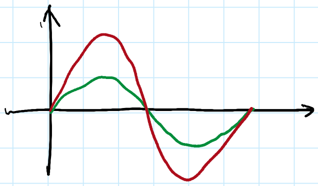
We could simply put a comparator on the shaper output. However, this is not very precise, as a comparator triggered by a fixed threshold would trigger at slightly different shape "times" depending on the amplitude (and so on the energy) of the incoming photon



time difference on same photon arrival time

UNIPOLAR TIME COMPARATOR

To solve the amplitude dependence issue, we can use a bipolar comparator, that triggers at the zero crossing

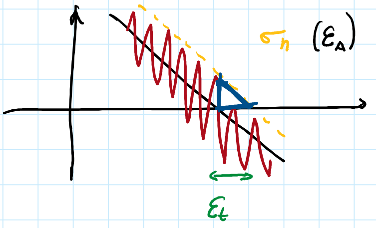


The zero crossing is not E dependent anymore

We can obtain this shape for example by differentiating the unipolar shape

BIPOLAR COMPARATOR

However also the timing precision is affected by noise. On the zero crossing the signal affected by noise



$$\varepsilon_t \cdot \left.\frac{dV}{dt}\right|_{V=0} = \varepsilon_A$$

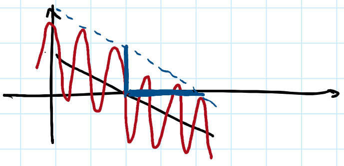
$$\varepsilon_t = \frac{\varepsilon_A}{\left.dV/dt\right|_{V=0}}$$

(linearizing at zero crossing

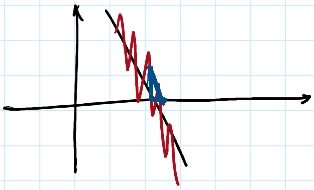
TIME JITTER RMS

We can observe that the time noise depend on the slope of the signal

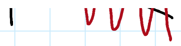
However the slope of the signal is dependent on signal energy itself.



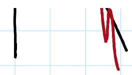
low E photons



high E photons



low E photons

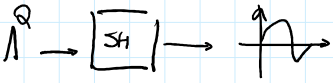


high E photons

x with the same voltage noise, the timing precision is better on a high energy photon

We can derive an alternative expression for time jitter considering

$$G_Q = \frac{V_{omax}}{Q}$$

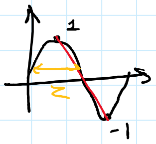


$$\varepsilon_T = \frac{\varepsilon_n}{\left(dV/dt\right)} \frac{G_Q}{G_Q} = ENC_{in} \cdot \frac{1}{\frac{1}{G_Q}\left(\frac{dV}{dt}\right)} = \frac{ENC_{in}}{\frac{Q}{V_{omax}} \cdot dV/dt} =$$

$$= \frac{1}{SNR|_Q \; \frac{dV'}{dt}}$$

⟶ derivative normalized to 1

$$\frac{dV}{dt} \cdot \frac{1}{V_{max}}$$



$$\Rightarrow \frac{\Delta V'}{\Delta t} = \frac{2}{2\tau} = \frac{1}{\tau}$$

↑ tau of bipolar gaussian shape

$$\varepsilon_T = \frac{ENC|_Q}{Q} \cdot \tau$$

RAMO THEOREM

Saturday, July 6, 2024 10:26 AM

SIGNAL FORMATION

The shockley Ramo theorem states a relation between

INDUCED CURRENT ON AN ELECTRODE $\Leftrightarrow$ $\frac{d}{dt}$ (RATE OF CHANGE) OF ELECTROSTATIC FLUX ($\vec{E}$ field on electrode surface)

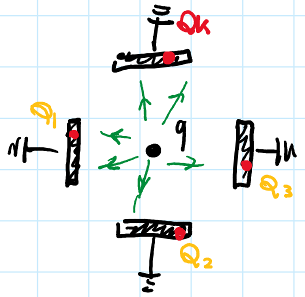
$\downarrow$

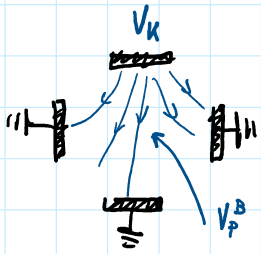
this can lead us to a relation between induced current on electrode and the istantaneous position (and movement) of moving charges

RAMO

We start by considering the green reciprocity theorem. Considering two cases in the same enviroment

$\ast$ derived from ∇ poisson equation





A

- Ground all electrodes
- Charge is placed in point P

$\Downarrow$

the P charge generates an electric field, attracting charges on electrodes

on A

q_P V_P
Q_K V_K
Q_1 V_1

B

- ground all electrodes, except K
- No charge
- apply voltage on intrest electrode

$\Downarrow$

Voltage will be present on the point P

on B

0 V_P
Q_K V_K

Q_K V_K
Q_1 V_1
Q_2 V_2
Q_3 V_3

0 V_p
Q_K V_K
Q_1 V_1
Q_2 V_2
Q_3 V_3

=> Green reciprocity theorem states that

$$\sum Q_i^A V_i^B = \sum Q_i^B V_i^A$$

$$\begin{bmatrix} q \\ Q_K^A \\ Q_1^A \\ Q_2^A \\ Q_3^A \end{bmatrix} \begin{bmatrix} V_p^B \\ V_K^B \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ Q_K^B \\ Q_1^B \\ Q_2^B \\ Q_3^B \end{bmatrix} \begin{bmatrix} V_p^A \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving the equation for our situation

$$q_p V_p^B + Q_K^A V_K^B = 0$$

particle charge

$$Q_K^A = -q_p \frac{V_p^B}{V_K^B} = -q_p \tilde{V}_w$$

normalized potential at point p induced by 1V on V_K^B electrode

potential at point p induced by electrode at V_K^B (no charge)

induced charge on plate K due to q_p

potential inducing V_P^B on electrode

GREEN RECIPROCITY

To derive the induced current on electrode K we can use the definition of current

$$i_K(t) = \frac{dQ_K}{dt}$$

Following green reciprocity

$$i_K(t) = \frac{dQ_K}{dt} = -\frac{d(q_p \tilde{V}_w)}{dt} = -q_p \frac{d\tilde{V}_w}{dt} \cdot \frac{d\vec{\ell}}{d\vec{\ell}} = -q_p \frac{d\tilde{V}_w}{d\vec{\ell}} \frac{d\vec{\ell}}{dt}$$

$$\vec{E} = -\frac{dV_w}{} \qquad \frac{d\vec{x}}{} = \vec{v}$$

$$\vec{E}_w = -\frac{dV_w}{d\vec{x}} \qquad \frac{d\vec{x}}{dt} = \vec{v}$$

along line ℓ

$$i_k(t) = q_p \vec{E}_w \cdot \vec{v}$$

scalar product

weighting potential field: not the true $\vec{E}$ field in the device, just the field induced by testing potential 1V on electrode

carrier velocity

if we assume drift in the device, we see that the q_p charge velocity is dependent on the true electric field present in the device

$$\vec{v} = \mu \vec{E}$$

(mobility model)

RAMO INDUCED CURRENT

The induced charge can be calculated on the weighting potential map

$$Q_i(t) = \int_0^t i(t)\,dt = \text{by RAMO} = \int_0^t q\vec{E}_w \vec{v}\,dt$$

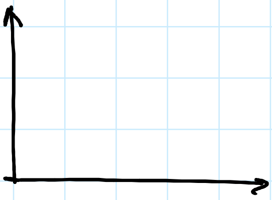
$$= \int_{z_0}^{z} q\vec{E}_w \cdot d\vec{x} = -q\left[V_w(z) - V_w(z_0)\right]$$

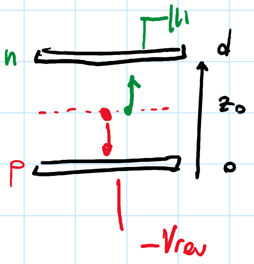
line integral

INDUCED CHARGE

SIGNAL FORMATION EXAMPLES

Assuming a device with ~ uniform field (like pin junction) we can derive using ramo the signal generated from the detector when a charge moves in it generated from the detector when a charge moves in it





Wheigthening field

$$E_W = -\frac{dV_W}{dx}$$

constant on device

$$= \frac{V_W}{d} = \frac{1}{d}$$

currents

$$i_n(t) = -q(-E_W \cdot V_n) = q\frac{1}{d}\cdot V_n = \frac{q}{T_n}$$

$$i_p(t) = q(-E_W V_n) = q\frac{1}{d}V_p = \frac{q}{T_p}$$

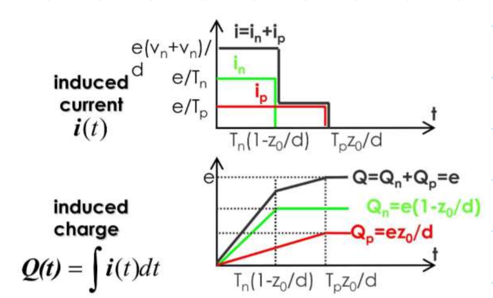
Velocity

$$V = \mu E = \mu\frac{V_{rev}}{d}$$

current depend on transit time in the WHOLE ELECTRODE however carriers are not transiting for the whole time, as they are generated in position z_0

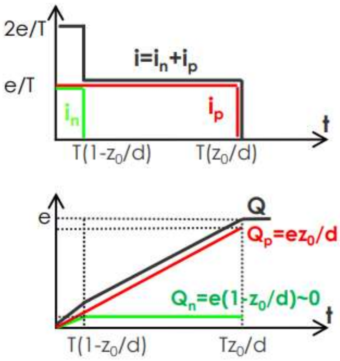
$$\begin{cases} i_n(t) & 0 < t < T_n\left(1-\frac{z_0}{d}\right) \\ i_p(t) & 0 < t < T_p\left(z_0/d\right) \end{cases}$$

Consider that typically also, $V_p < V_n$ as holes have a lower mobility (so lower velocity)



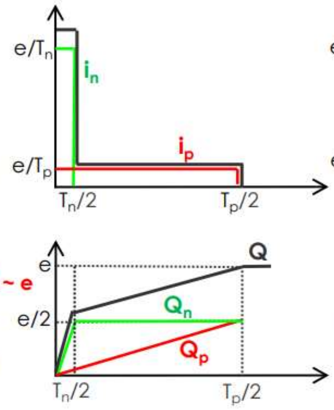
- sensing the charge moving
- not important collected charge
- at the end $Q = q$ (collected charge)

z_0~d (anode side)
$v_n=v_p=v$



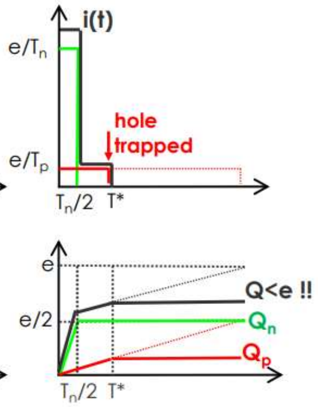
- equal currents
- hole induction dominant

z_0=0.5d
v_n>>v_p



- i_n>>i_p but equal contribution to ind. charge
- total induction after slow hole collection

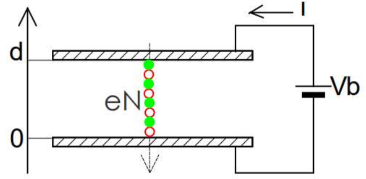
as before, but
hole trapped @ $t=T^*$



- signal loss !!
- total induced charge dependent on trap position

✓*Induced current (charge) in planar electrode geometry*

Continuous ionization

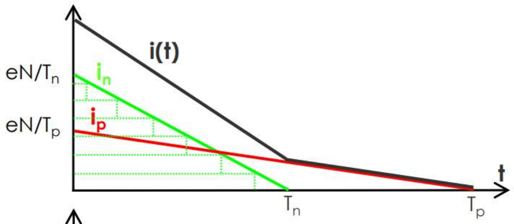


$$\boldsymbol{E}_w = -\frac{1}{d}\boldsymbol{k}$$ **weighting field**

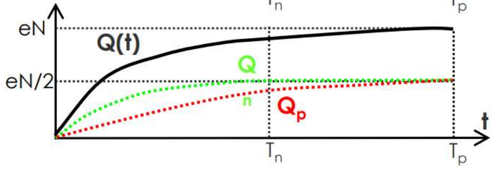
$$i_n(t) = eN\frac{v_n}{d}(1-t/T_n) = \frac{eN}{T_n}(1-t/T_n) \qquad 0 \le t \le$$

$$i_p(t) = eN\frac{v_p}{d}(1-t/T_p) = \frac{eN}{T_p}(1-t/T_p) \qquad 0 \le t \le$$

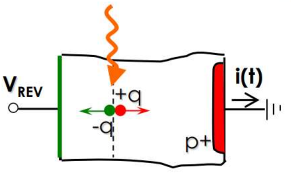
induced current

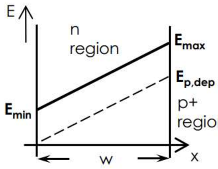


induced charge



✓*Induced current (charge) in pn junction*

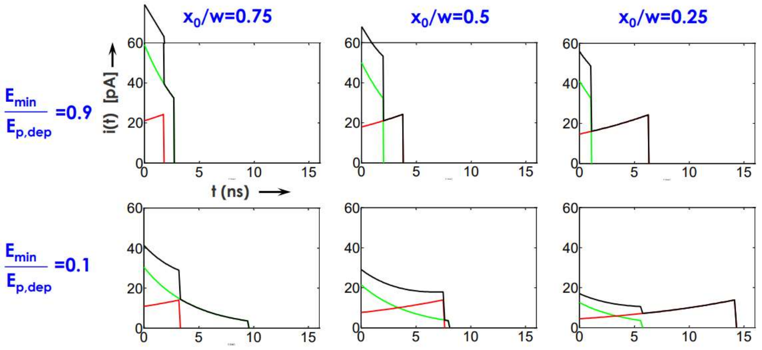




induced currents

$$i_h(t) = \frac{q}{w}\mu_h\left(E_{\min} + \frac{qN_D}{\varepsilon}x_0\right)e^{\mu_h q\frac{N_D}{\varepsilon}t} \qquad 0 \le t \le t_h$$

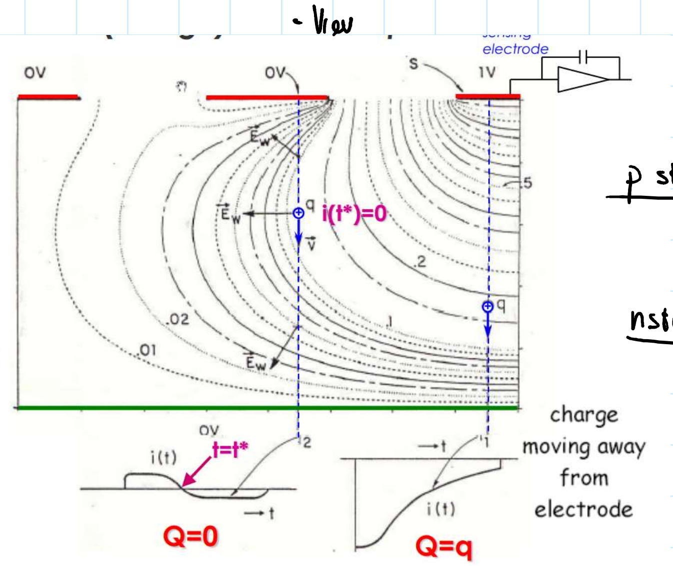
$$i_e(t) = \frac{q}{w}\mu_e\left(E_{\min} + \frac{qN_D}{\varepsilon}x_0\right)e^{-\mu_e q\frac{N_D}{\varepsilon}t} \qquad 0 \le t \le t_e$$



Silicon
w=300μm
$N_D=10^{12}$ cm^{-3}

E_{peak} @V_{dep} =
=qwN_D/ε =
=4800 V/cm

- View



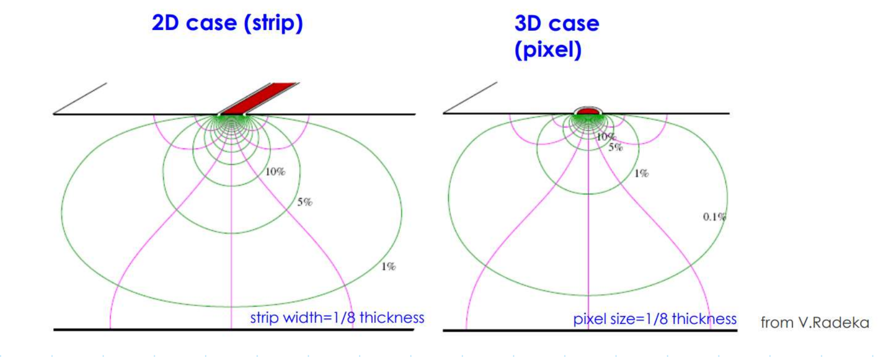
p strip

nstrip

$i = q \vec{E_w} \cdot \vec{v_w}$ Stripe detection mechanism

here if we can observe 2 things

- a charge detected by one strings can influence the current on another electrode
- the strip width - comparable with detector depth - makes the Ew weightening field evenly distributed
  => charge collected on detector depends on starting position of ionization



from V.Radeka

We can reduce both effects by having a strip width << detector depth

↳ Weightening Potential will be concentrated only near the strip

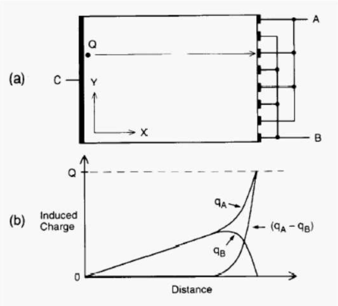
→ Weightening Potential will be concentrated only near the strip

=> Charge collection will be almost independent on initial ionization position :

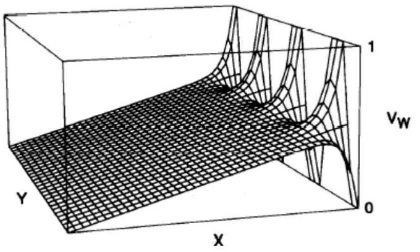
$Q \sim q$ (as if it started from one electrode to another

=> also almost independent for hole trapping

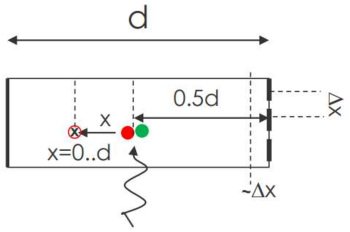
DETECTOR STRIPS



P.Luke, IEEE Trans Nucl. Sci., vol.42, no.4, Aug. 1995

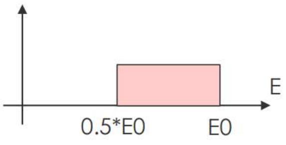


- Two inter-digitated coplanar grid electrodes sense the motion of charge carriers in the detector (solid-state equivalent of the “Fritsch grid” of gas detectors)
- A small potential difference applied btw the C-grid and NC-grid to avoid charge sharing and double polarity signals
- When generated in the bulk, a charge carrier induces equal amount of charge on the 2 grids. A net difference signal is induced only when the carriers to be collected (e.g. electrons) are close to the coplanar electrodes.
- The net result is a measured charge nearly independent of the interaction depth

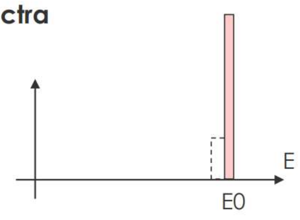


- radiation incident at x=0.5d.
- hole trapping can take place at x, uniformly distributed within (0, 0.5d)
- electrons travel with no trapping

energy spectra



single-ended readout

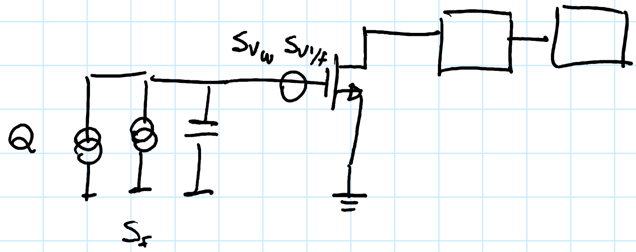


differential readout

ADVANCED SILICON DRIFT DETECTORS

Sunday, July 7, 2024   4:53 PM

Detector capacitance plays a key role in ENC



series white

$$ENC^2 = A_1 \frac{2KT\alpha}{\omega_t} C_D \cdot \overbrace{\left(\sqrt{\frac{C_G}{C_D}} + \sqrt{\frac{C_D}{C_G}}\right)^2}^{=4} \frac{1}{\tau}$$

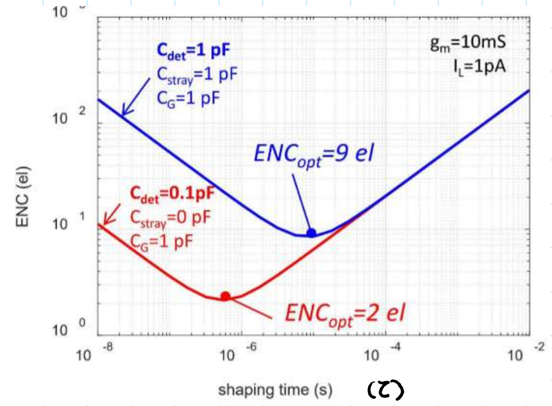
↑ f cutoff MOSFET

parallel white

$$+ A_2 (\pi A_f C_G) C_D \underbrace{\left(\sqrt{\frac{C_G}{C_D}} + \sqrt{\frac{C_D}{C_G}}\right)^2}_{=4} + A_3 (q I_L) \tau$$

series 1/f

when matching cap



(τ)

Supposing we can optimize setting $C_{Gate} = C_d$

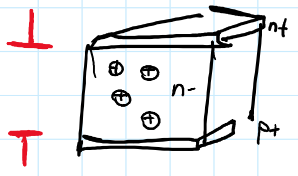
=> C_d determines contribution of serien white and 1/f

=> ENC^2_{opt} (find by setting white par = white sere) (τ)

$$\propto \sqrt{\frac{C_D I_L}{\omega_t}}$$

Lowering as much as possible C_D increases SNR

DETECTION CAPACITANCE

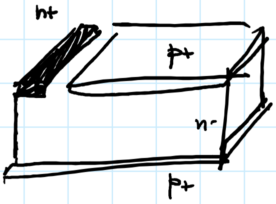


classic pin junction

- A of capacitance is big
- tickness cannot be increased due to technology limitations

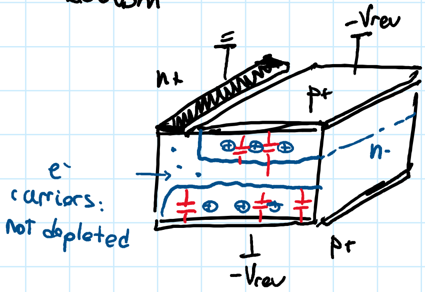
- thickness cannot be increased due to technology limitations (wafer thickness)
  - → would increase active area
  - → would decrease capacitance

CLASSICAL PIN LIMITATIONS

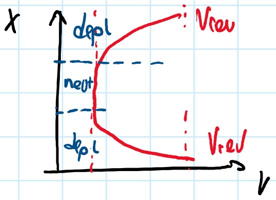


To increase active we could develop the pin horizontally

the depletion region would develop between p+ and n- (like in classic pin junctions) both on top and bottom



This however stills results in a big depletion capacitance (big active area on both sides)



no $\vec{E}$ field in the neutral region

⇒ However, to increase performance, we can deplete the whole n- thickness

⇒ this gives a huge advantage in capacitance as in fact reduces active area of the depletion region

The active area now becomes basically only the fringing contribution

we must deplete all the device

$$V_{rev} = \frac{qN_D}{2\varepsilon_0\varepsilon_s} \underset{\uparrow}{x_d^2} = \frac{qN_D}{2\varepsilon_0\varepsilon_s}\left(\frac{d}{2}\right)^2 = \frac{1}{4}\frac{qN_D}{2\varepsilon_0\varepsilon_s} d^2$$

both sides → half thickness

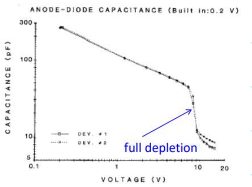
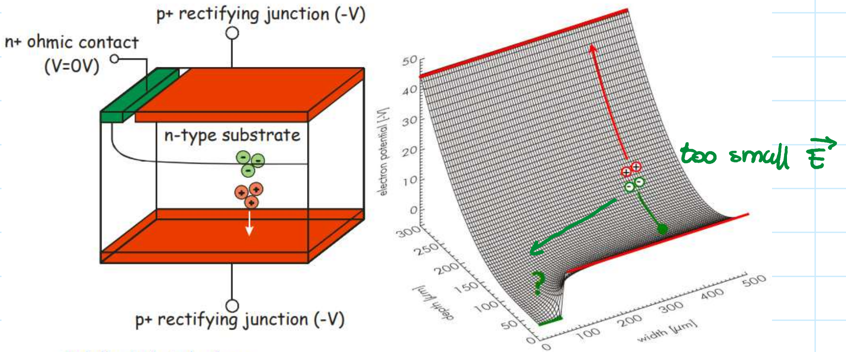


Fig. 12. Capacitance versus voltage plots of two of the test devices provided on the wafer for monitoring its doping uniformity.

(EMILIO GATTI) SILICON DRIFT DETECTOR - FULL DEPLETION

However this design present an issue.
The drift between the two contacts is too slow
as the field bends only toward the center

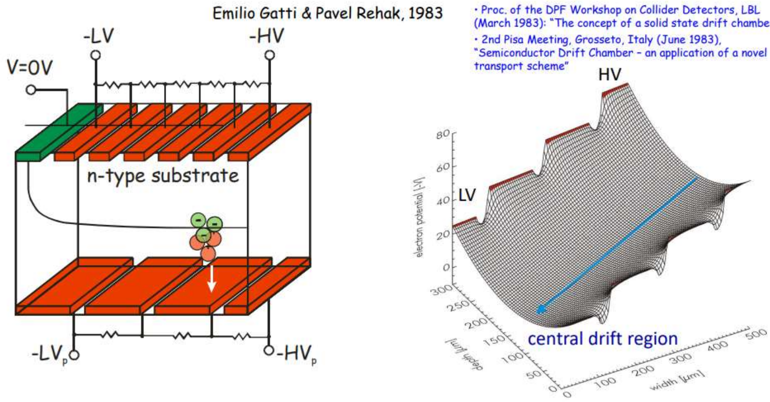


☺ fully depleted volume
☺ capacitance value has been decoupled from detector active area
☺ holes are (quickly) collected by the p+ electrodes
☹ How to bring electrons toward the anode?

pn COLLECTION DRIFT

Emilio Gatti & Pavel Rehak, 1983

- Proc. of the DPF Workshop on Collider Detectors, LBL (March 1983): "The concept of a solid state drift chamber"
- 2nd Pisa Meeting, Grosseto, Italy (June 1983), "Semiconductor Drift Chamber – an application of a novel transport scheme"



- p+ junctions segmented in strips with linearly increasing potential on both sides
  - → uniform drift field parallel to the surface
  - → signal electrons are focused in the center of the wafer and drift at constant velocity towards the readout anode

Emilio Gatti & Pavel Rehak, 1983

- Proc. of the DPF Workshop on Collider Detectors, LBL (March 1983): "The concept of a solid state drift chambe
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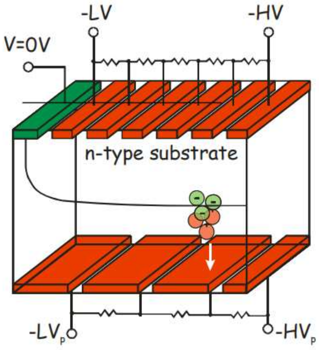
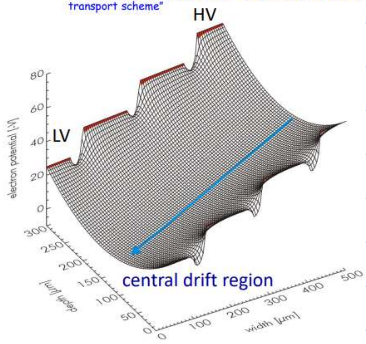
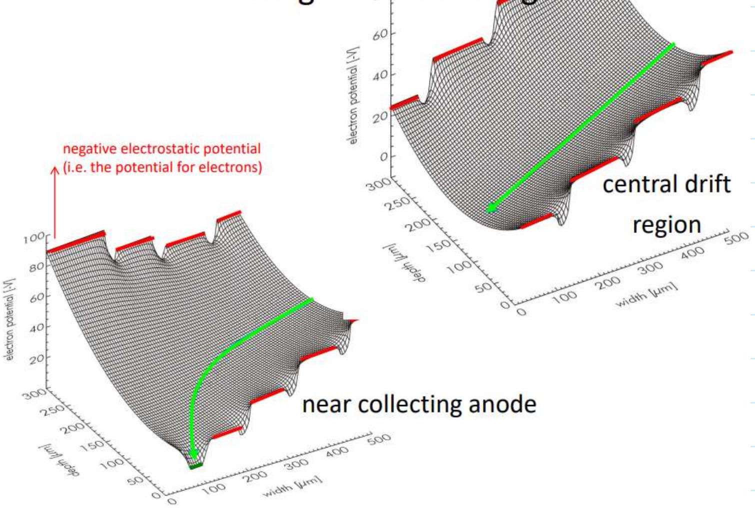




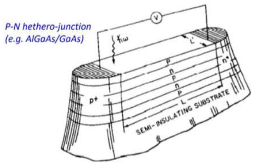
- p+ junctions segmented in strips with linearly increasing potential on both sides
  - → uniform drift field parallel to the surface
  - → signal electrons are focused in the center of the wafer and drift at constant velocity towards the readout anode

Shape the potential for the charge collection region



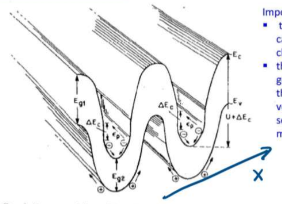
The simultaneity of ideas

A novel avalanche photodiode (APD) concept, "the channeling APD"



- Using a new interdigitated p-n junction structure, electrons and holes are spatially separated and impact ionize in layers of different band gap.
- Thus the effective ionization-rates ratio can be made extremely high (α/β > 100), while maintaining a high gain,

Fig. 1. Schematic diagram of the APD (not to scale). The p-layers have a wider band gap than the n-layers. See text for layer thicknesses ai



Important features:

- the unique capacitance-voltage characteristic
- the interdigitized geometry which allows the depletion of large volumes of semiconductor materials

Fig. 3. Band diagram of the APD under operating conditions. ϵ_p is the parallel electric field causing carriers to impact ionize. U is the sum of the punchthrough voltage and the built-in potential. The valence-band discontinuity has been assumed negligible with respect to ΔE_c. Shown is also the electron-hole separation mechanism.

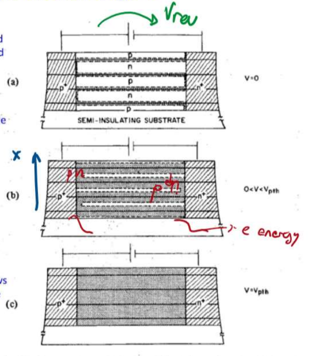


Fig. 2. (a) Cross section of the interdigitated p-n junction device at different reverse-bias voltages V. (b) The shaded portions of the layers represent the depleted volume which increases with increasing bias. (c) V_{pth} is the voltage at which the layers are completely depleted. Any further increase in bias adds a constant field ϵ_p parallel to the device length. Note that to ensure a parallel electric field constant along most of the device length L, this dimension is made much greater than the layer thicknesses. For simplicity of illustration equal doping levels (p = n) have been assumed.

F. Capasso, IEEE Trans. Electron Device, Vol. ED-29, No.9, Sept. 1982

SILICON DRIFT DETECTOR

SPECIALIZED SDD

- SPECTROSCOPY ORIENTED

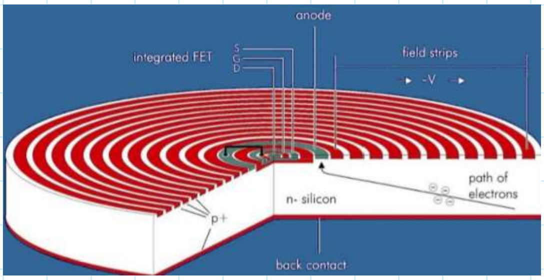
these SDD do not implement position sensing.

- SPECTROSCOPY ORIENTED

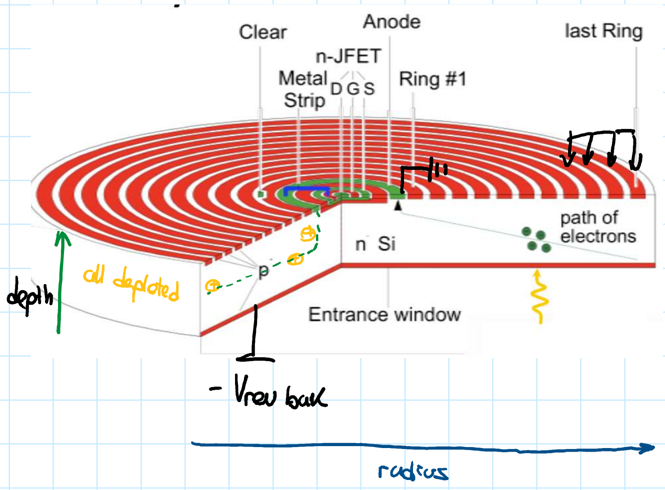
  these SDD do not implement position sensing. they use all their active area to collect photons and distinguish between different impacting photon energies
- POSITION SENSING

  These devices detect the impact point like a camera constructing an image (2D - 3D - 1D too) they can or cannot distinguish different energies of impact photons (pixel spectroscopy)

SPECTROSCOPY

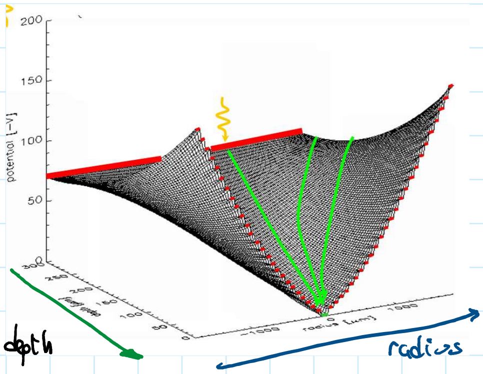


- cylindrical shape : single anode in the center
  - ↳ all active area to detection
- Back contact is the exposed side to photons
  - ↳ p+ back contact is an entrance window
- Transistor is integrated in the detector, directly connected to the anode



p+ front segmentation : makes the electrons drift toward the center

p+ back
p+ front
: serves to fully deplete the SDD (- cap) can divide the carriers toward the center depth



A typical = 5-100 mm²

Det cap = 100-200 fF

CYLINDRICAL SDD

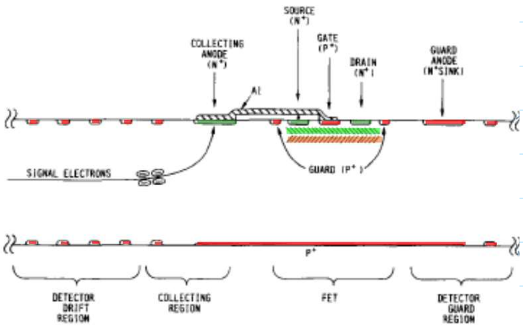
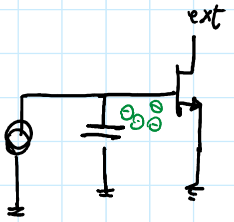


Fig. 1. Qualitative cross section of a completely depleted detector (semiconductor drift detector) with the input FET integrated on it.



the p+ guards protect the electron from being collected in the source of the JFET (at ground too) or drain at Vso

=> electrons are collected in the anode, connected to the gate of the JFET via a short metal contact

↳ extremely low capacitance on preamp

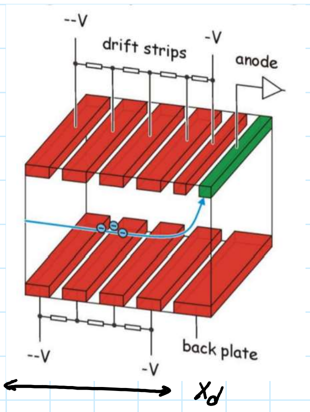
↳ smaller contributions on series noises (dependant on capacitance)

The cylindrical shape enables a 360° collection

POSITION SENSING SDD

Position sensing can be achieved

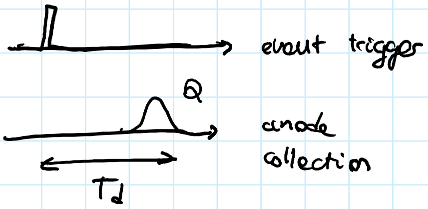
- via subdivision of strips in anode
- via timing correlation



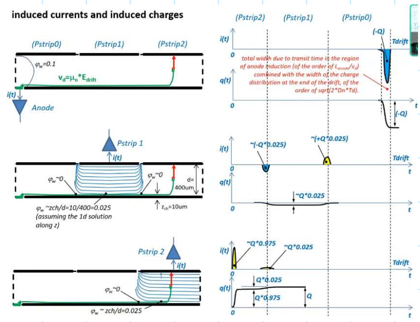
Assuming
- constant Electric field
(in low doped n- $\frac{dE}{dx} \sim 0$)

=> constant drift velocity
$v = \mu E$
(or due to sizing of Vsaturation in whole device)

$X_d = V_{drift} \cdot T_d$



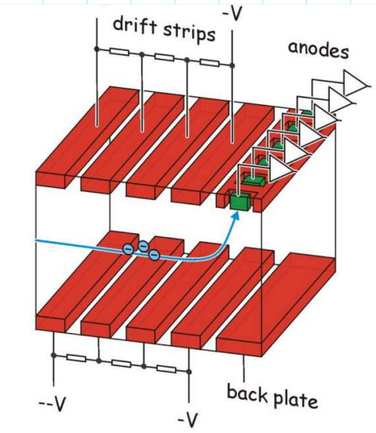
(Even if the relation is not so liner, it can be derived from testing)

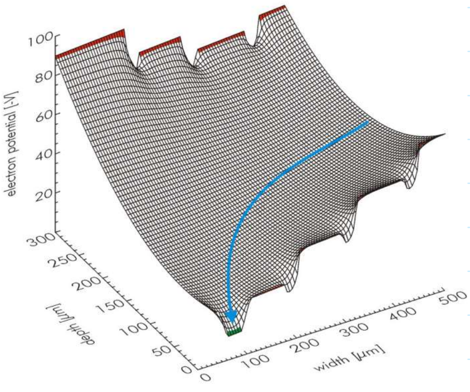

induced currents and induced charges

total width due to transit time in the region of anode induction (of the order of L_{anode}/v_d) combined with the width of the charge distribution at the end of the drift, of the order of sqrt(2*Dn*Td).

Need of a start event trigger
- available if hit time is known - a priori (particle accelerators)
- can be derived by holes generation; immediately collected by p+ strips

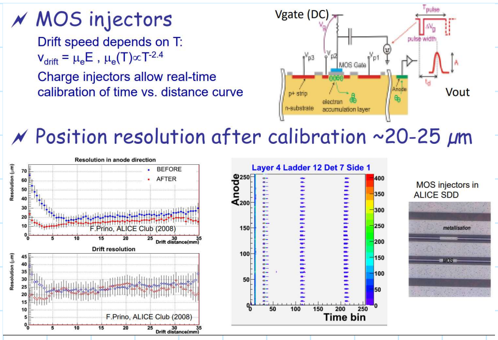
2D POSITION SENSING W TIMING





2D position sensing can be achieved combining multiple strips on one direction and timing detection on the other

2D DETECTION



- MOS injectors
  - Drift speed depends on T:
  - $v_{drift} = \mu_e E$, $\mu_e(T) \propto T^{-2.4}$
  - Charge injectors allow real-time calibration of time vs. distance curve
- Position resolution after calibration ~20-25 μm

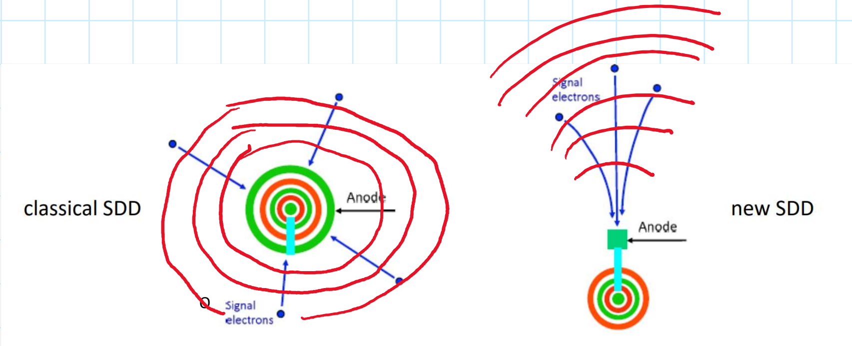
calibration mechanisms for timing detection.

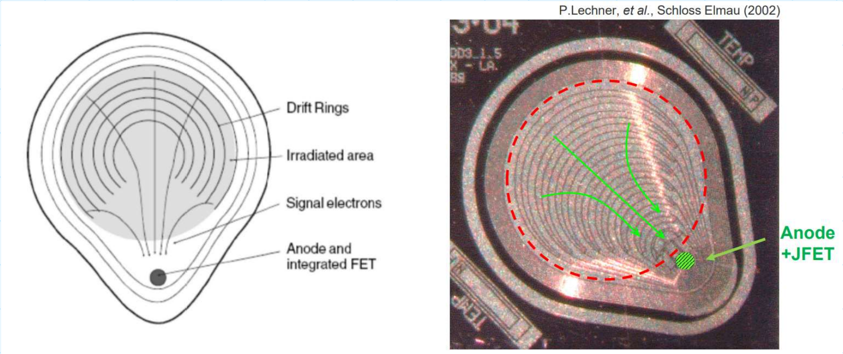
ADVANCED SDD CONCEPTS

the cylindrical SDD needs a circular anode
=> big capacitance

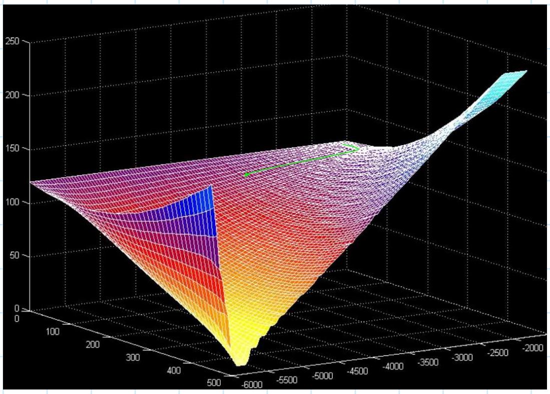
If we instead of making the device circular, but we instead make it drop-like

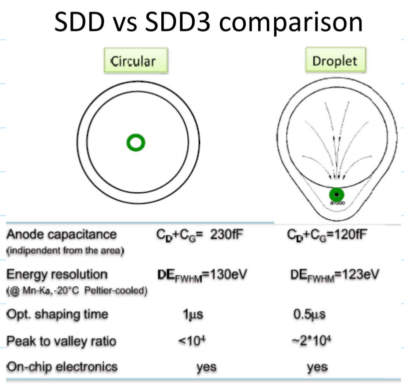
drift strips




P.Lechner, *et al.*, Schloss Elmau (2002)

This can optimize the area of the anode and the JFET making it less and optimizing for ENC.

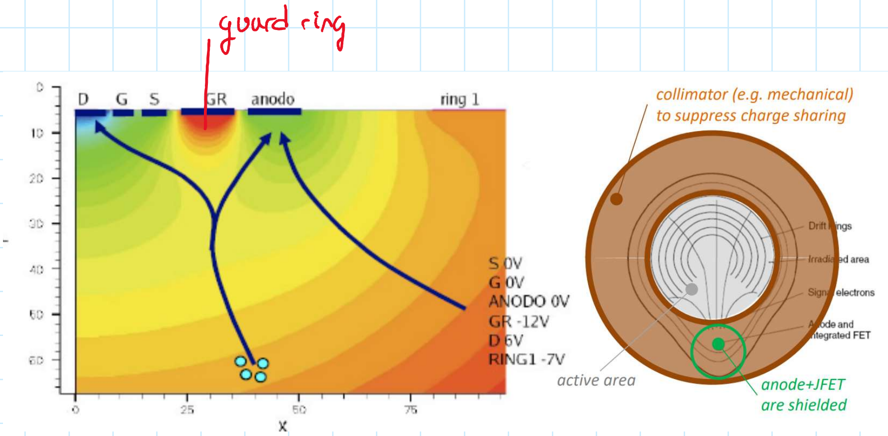




SDD vs SDD3 comparison

	Circular	Droplet
Anode capacitance (indipendent from the area)	C_D+C_G= 230fF	C_D+C_G=120fF
Energy resolution (@ Mn-Ka, -20°C Peltier-cooled)	DE_{FWHM}=130eV	DE_{FWHM}=123eV
Opt. shaping time	1µs	0.5µs
Peak to valley ratio	$<10^4$	$\sim 2*10^4$
On-chip electronics	yes	yes

DROPLET SDD SDDD



If a interaction (photon hit) happens near the anode region, it may happen that the interaction charge is splitted above the guard ring an some can pass directly to the drain

=> this can be avoided putting a screen on the outer ring, avoiding collisions near the anode

↳ beware that the anode region will not be active area.

GUARD RING - NEAR ANODE COLLECTION

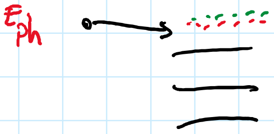
* Silicon Escape spectroscopy
- important to signal formation and radiation too.

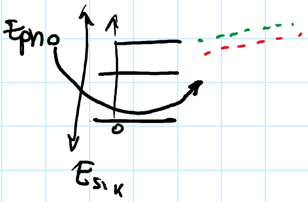
When doing spectroscopy, we are correlating the charge sensed via the electronics created by

When doing spectroscopy, we are correlating the charge sensed via the electronics created by the incoming photon via the mean creation energy

$\langle N\rangle = \langle w\rangle E_{ph}$

However, it can happen thath incoming high energy photons interact with the lower shells of silicon



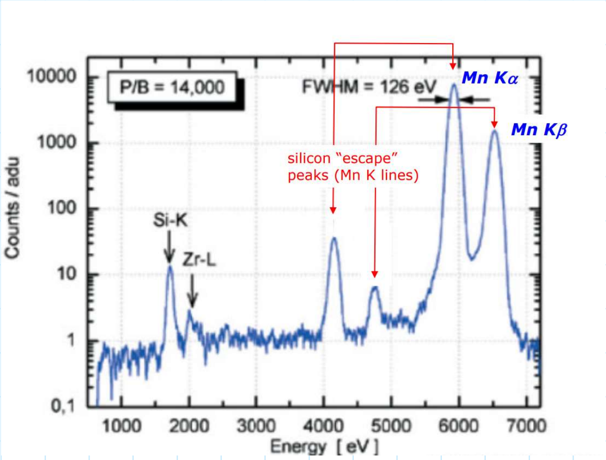


$\langle N_D\rangle = \langle w\rangle E_{ph}$

$\langle N_D\rangle = \langle w\rangle\cdot(E_{ph} - E_{Si\,K})$

This result in "Fake" incoming detecteded photons at energies at exactly

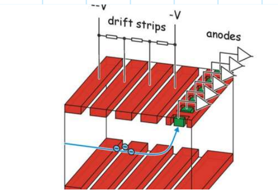
$E_{ph} - E_{K\,si}$



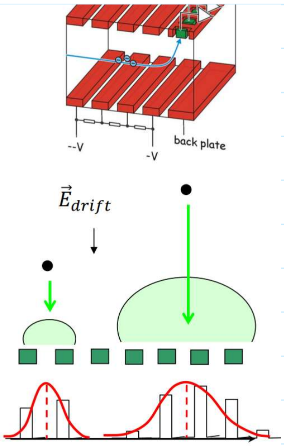
SILICON ESCAPE PEAKS

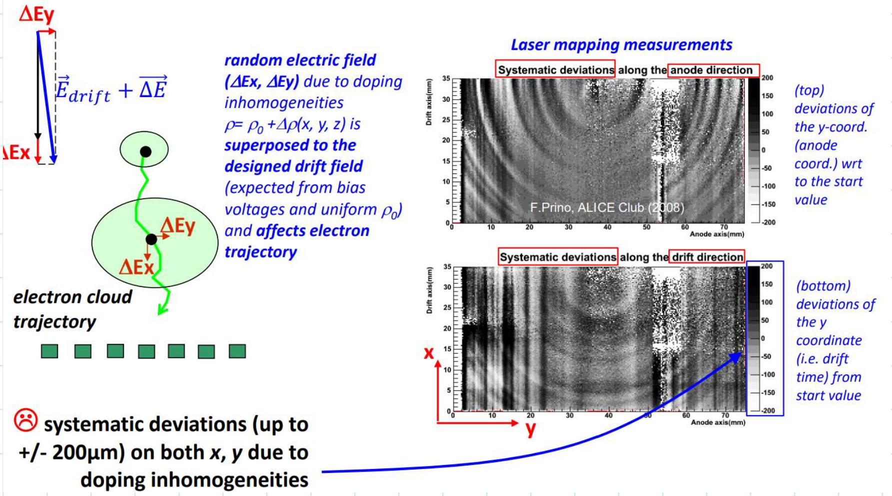
2D position sensing in MULTI ANODE SDD presents some limitation

- timing direction
  ⇒ a trigger is needed
- strips direction



- strips direction
  => drift adds a non equal distribution of the charge cloud depending on the timing direction position
  => drift direction is also not perfectly vertical due to systematic process errors





random electric field ***(ΔEx, ΔEy)*** *due to doping inhomogeneities* $\rho = \rho_0 + \Delta\rho(x, y, z)$ *is* ***superposed to the designed drift field*** *(expected from bias voltages and uniform* ρ_0*) and* ***affects electron trajectory***

Laser mapping measurements

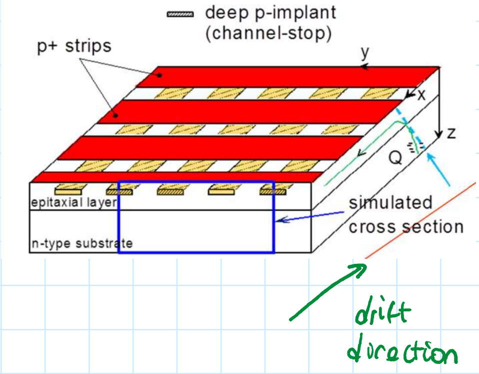
(top) deviations of the y-coord. (anode coord.) wrt to the start value

(bottom) deviations of the y coordinate (i.e. drift time) from start value

☹ **systematic deviations (up to +/- 200µm) on both *x, y* due to doping inhomogeneities**

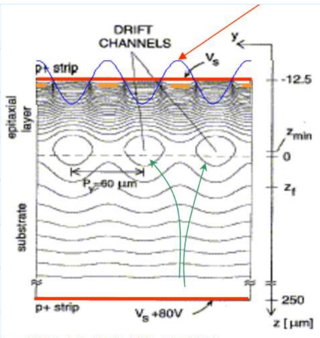
LIMITATIONS OF MULTI ANODE STRIP DET.

Suppression of lateral broadening with channel-stops

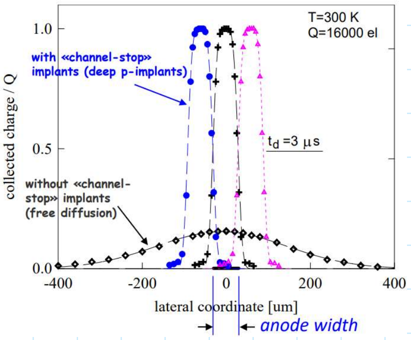


The p channel implant are put at a higher (negative, in module) potential to reject electron

In this way electrons drift in this channels avoiding dispersion in a charge cloud

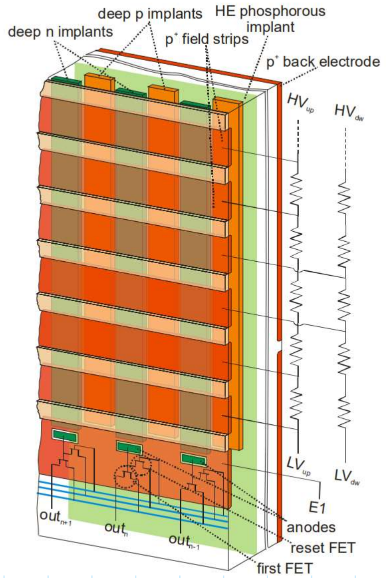


Castoldi, P.Rehak, P.Holl, NIM A377 (1996)



experimental results

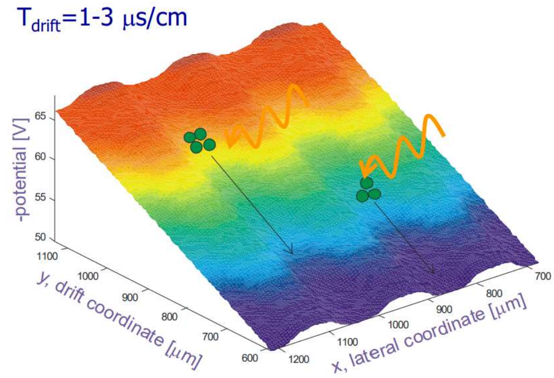
These devices are called Multi Linear SSD



- fully depleted n-type bulk
- p+ entrance window implanted on the back side
- array of p+ strips implanted on the front side
- channel-stops (deep p-implants) for lateral confinement
- channel-guides (deep n-implants) for lateral confinement and drift enhancement
- HE n implant locates the drift channel close to the finely structured surface
- on-chip electronics (JFET in source follower configuration)

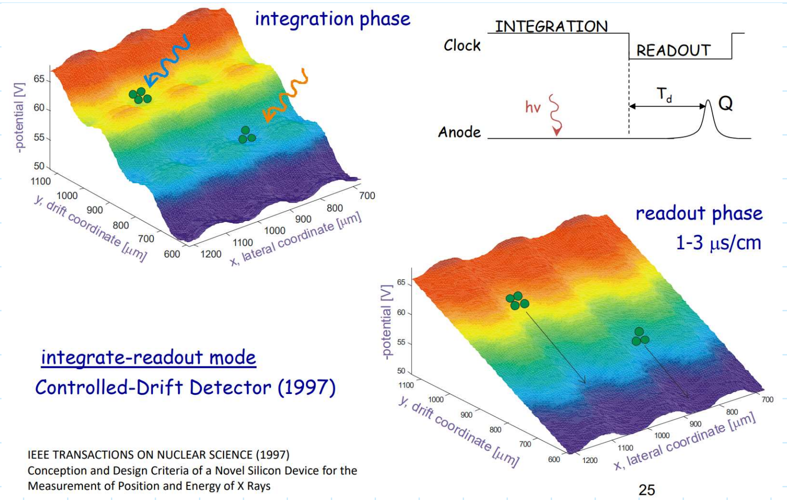
→ more doped so asymmetric potential on depth
=> n+ implants
=> drift line (minimum energy line) closer to surface

As before, trigger must
× external
× from holes to p+ immediate collection on strips/



MLSDD MULTI LAYER SDD

The MLSDD can also be used in another mode, as CDD (Controlled Drift Detector)


integration phase

readout phase
1-3 μs/cm

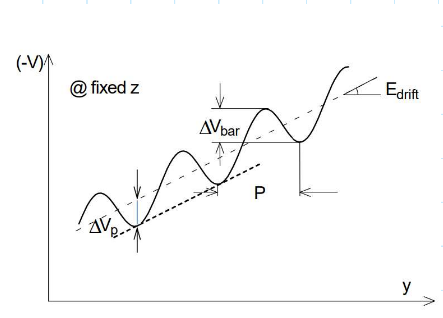
integrate-readout mode

Controlled-Drift Detector (1997)

IEEE TRANSACTIONS ON NUCLEAR SCIENCE (1997)
Conception and Design Criteria of a Novel Silicon Device for the Measurement of Position and Energy of X Rays

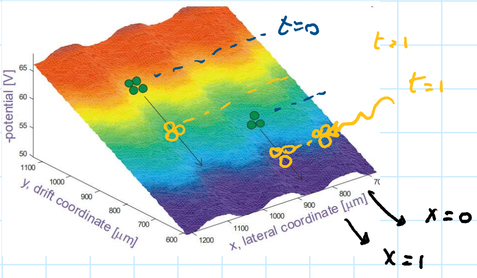
25

The voltage on the drift strips can be alternated during an "integration time" in which potential wells are created and electrons accumulate in it
When the voltage is then set to drift mode, all the integrated charge is collected via the classic readout



In the CDD however there is one problem
"out of time events"

- If during readout a photon collection happens it may seem from a different position from when it happened

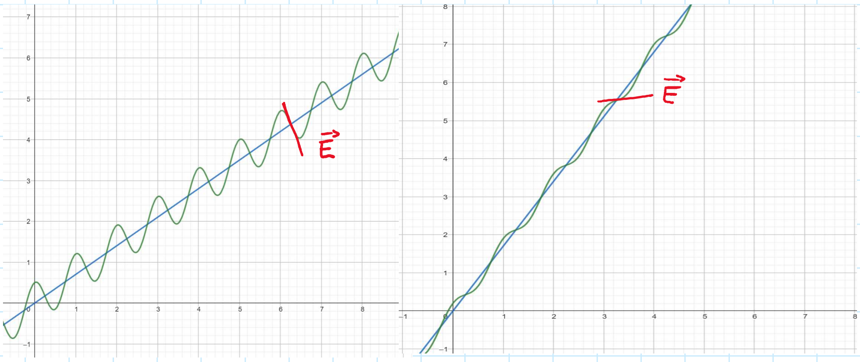


photon hit on x=0 happened on t=1 of drift time
=> it is detected as if it was collected at depth equal of the collection on x=1

To solve this problem we should stop the detection during readout time (maybe with a mechanical mask, not so easy)

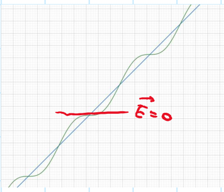
CDD CONTROLLED DRIFT DETECTOR

$V_{drift} = Ex$ $\qquad V_{pot} = V_{drif} + V_p \cos\left(2\pi \frac{x}{p}\right)$ ↳ pitch



$E = -\frac{dV}{dx} = 0$ $\qquad \frac{dV_{rift}}{dx} + V_p \frac{2\pi}{p} \sin\left(\pi \frac{x}{p}\right) = 0$

↓ max point



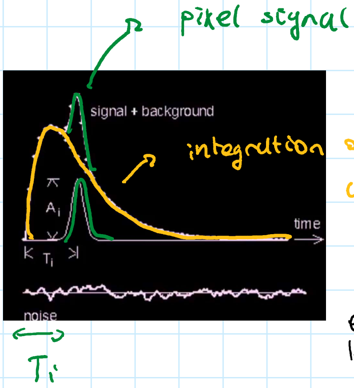
$E_{drift} + V_p \frac{2\pi}{p} (1) = 0$

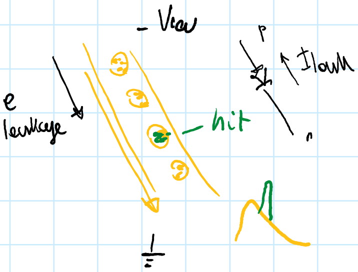
$V_p = \frac{E \cdot P}{2\pi}$

if $V_p > \frac{EP}{2\pi}$ => field inversion

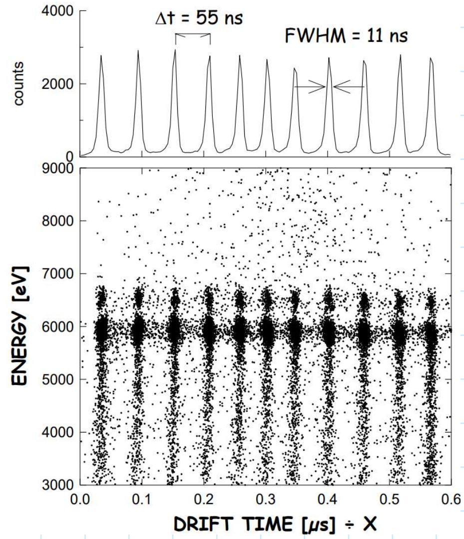
SIZING OF ΔV BARRIER

the integration - drift device has however a readout slighty different





BACKGROUND PEAK



correlating every reading with

- time
  => x (drift) position
- amplitude
  => collected charge (energy)

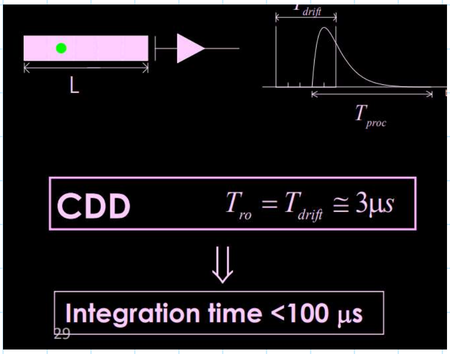
↓ we can obtain a 2D spectroscopy image!

(attention: in the integration time more photon hits could pile up! => may lead to "fake" high energies detected)

IMAGING SPECTROSCOPY

CDD are really fast as the to the whole readout of the image in one shot

CDD time ~ $T_{drift\ worst} + T_{shaper}$



CDD $T_{ro} = T_{drift} \cong 3\mu s$

⇓

Integration time <100 μs

29

Extremely fast respect to CCD detector!
(however works best with low level of occupancy)

CCD

CHARGE COUPLED DEVICES

A limitation of CCD is that the readout of the image happens pixel by pixel

$$CCD_{time} = N_{pix} \cdot \left(\frac{1}{f_{shift}} + T_{processing} \right)$$

- $1/f_{shift}$ is basically the drift time from 1 pixel to the next
- $T_{processing}$ (readout) is the τ of the output shape (shaper) optimized to reduce ENC

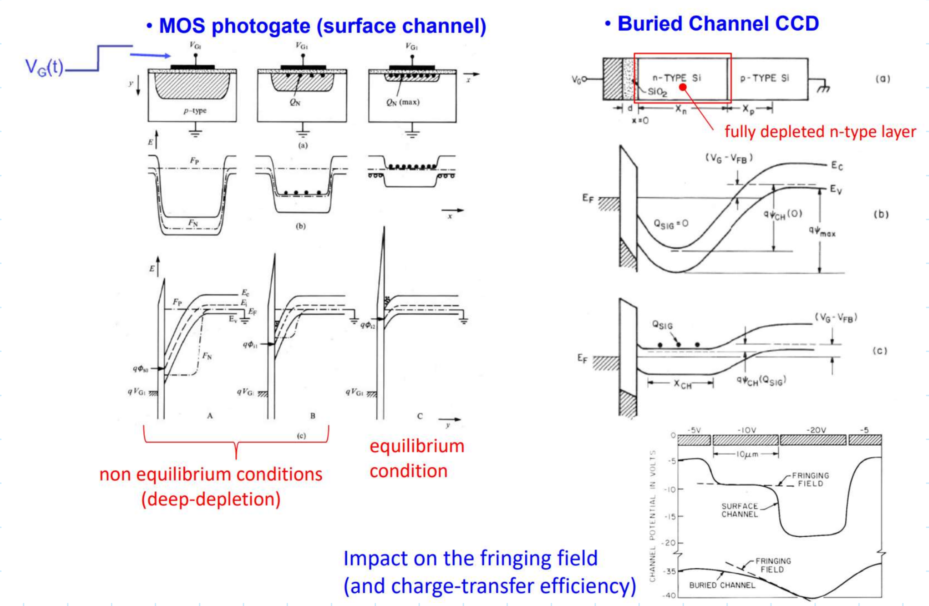
=> If integration time too large we increase leakage current shot noise

~~If we want higher frequencies (readout time less) we have to cut down processing time, increasing the noise~~

↳ or cool the device wildly!

higher frequency
higher resolution

CCD TIMING IS SLOW RESPECT TO CDD



• MOS photogate (surface channel)

• Buried Channel CCD

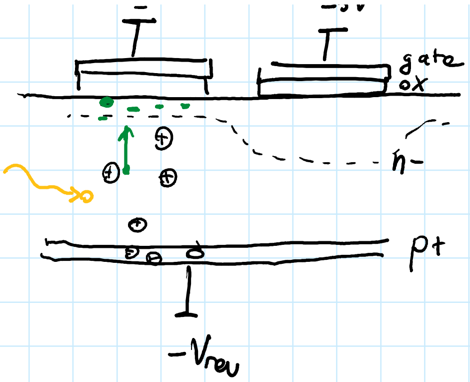
fully depleted n-type layer

non equilibrium conditions (deep-depletion)

equilibrium condition

Impact on the fringing field (and charge-transfer efficiency)

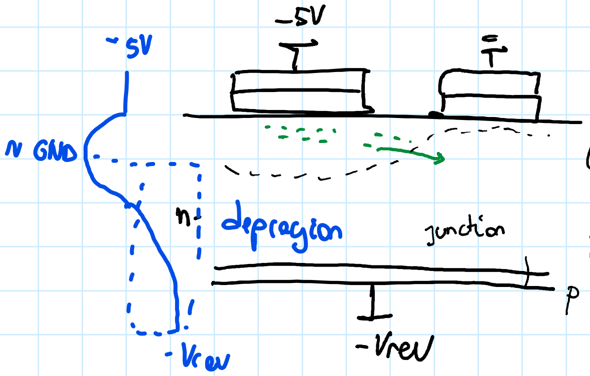
BURIED MOS



full depletion

the collected charge accumulates under the "active" CCD gate

the transfer happens moving the potential on the gates



(electrons move thawords higher potential)

CTE

charge transfer efficiency

- also drift in buried MOS happens ~ at the center depth of the channel
=> not near the surface

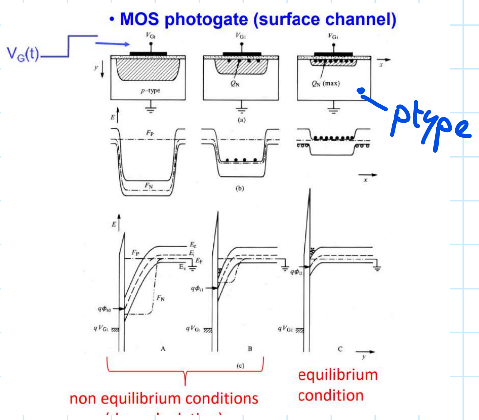
MOS PHOTOGATE

We could replace the pn fully depleted junction directly with the MOS capacitor

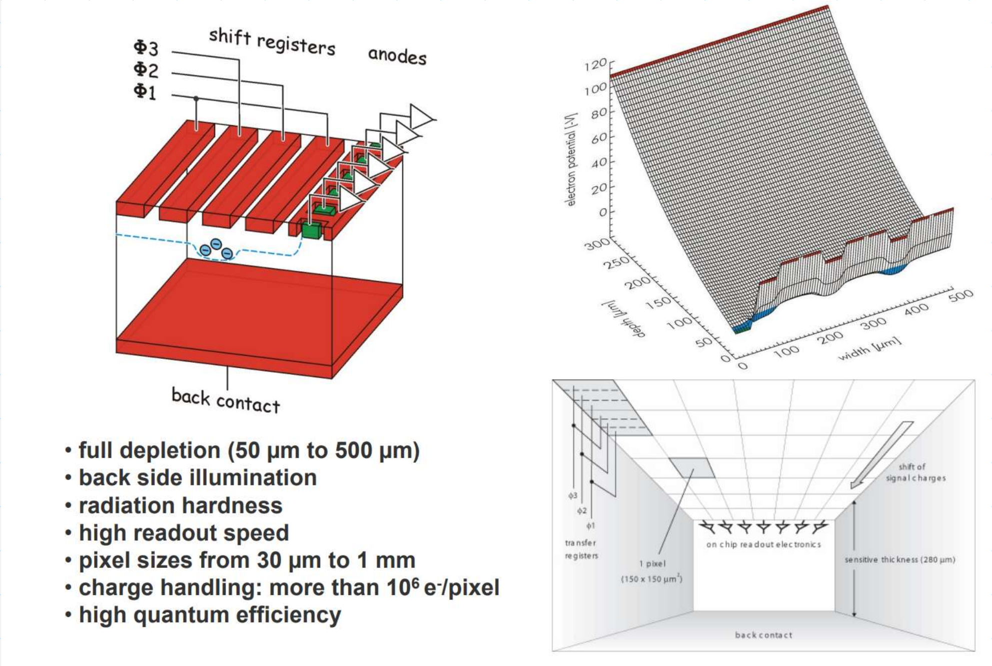
We can create a deep depletion region using the MOS field effect and collect the generated charge in the depletion region in the channel

then transfer it using the gate potentials like before

the transfer will happen near the MOS surface (near SiO$_2$)
=> not good because of trapping



MOS CCD TYPES

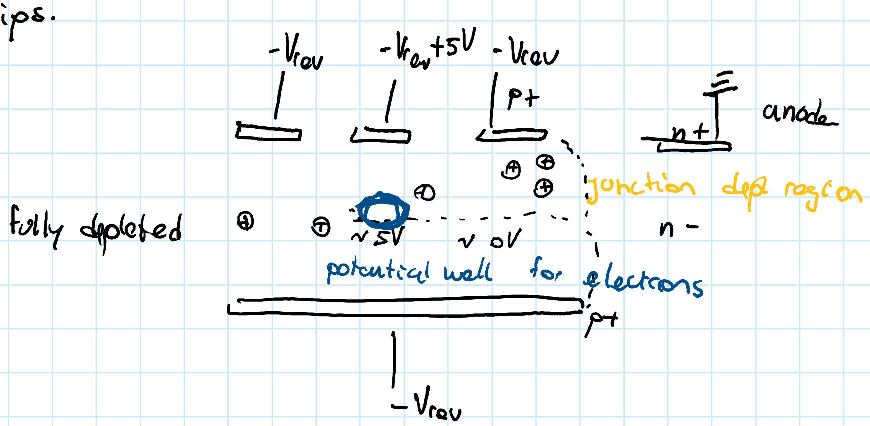


- full depletion (50 µm to 500 µm)
- back side illumination
- radiation hardness
- high readout speed
- pixel sizes from 30 µm to 1 mm
- charge handling: more than 10^6 e⁻/pixel
- high quantum efficiency

Another similar implementation of CCD can be done without a MOS capacitor.

Exploiting the already present bias p+ strips in SDD (Silicon Drift detectors)

Voltage can be modulated like in a CCD on those strips.

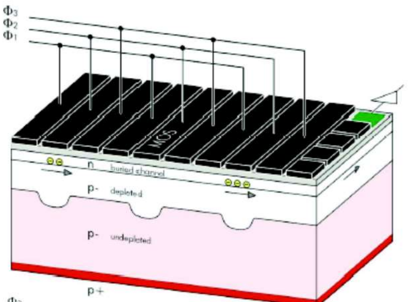


PN CCDs

pn-CCD vs. MOS-CCD

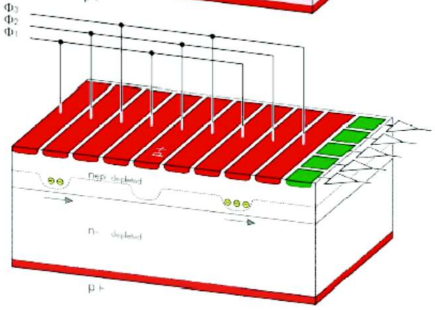
classic CCD structure

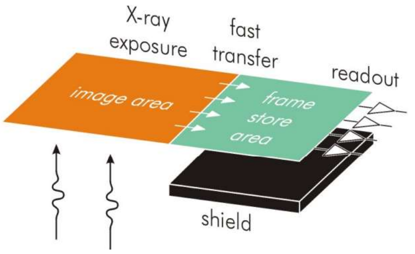
MOS CCD



- MOS transfer gates
  - ➢ pn junctions
- buried channel
  - ➢ deep transfer
- partial depletion
  - ➢ full depletion
- front-side illumination
  - ➢ backside entrance window
- serial readout
  - ➢ 1 preamp per channel

pn CCD





CCD readout can also have smarter configuration:

To allow integration time simultaneously to readout time, we can store the just integrated frame in another area for a readout

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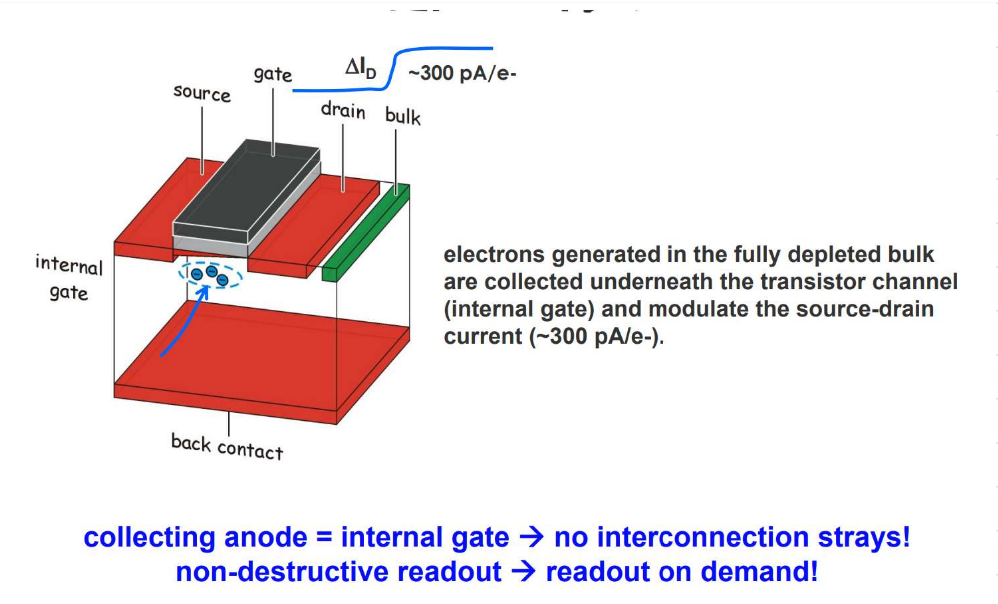
pn CCD vs MOS CCD

DEPFET



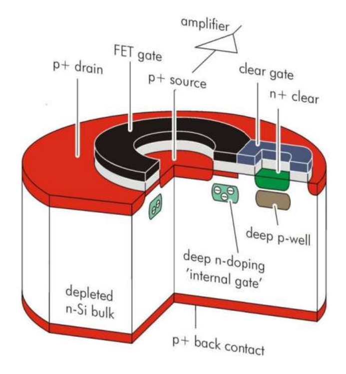
electrons generated in the fully depleted bulk are collected underneath the transistor channel (internal gate) and modulate the source-drain current (~300 pA/e-).

collecting anode = internal gate → no interconnection strays!
non-destructive readout → readout on demand!

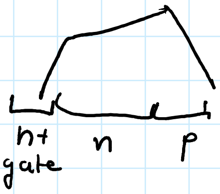
Instead of using MOSFETS for readout, we can use DEP-MOSFET.

The principle is similar to MOSFETs
Instead of having only the external gate DEP MOS also have an internal gate residing underneath the channel --> n+ internal gate

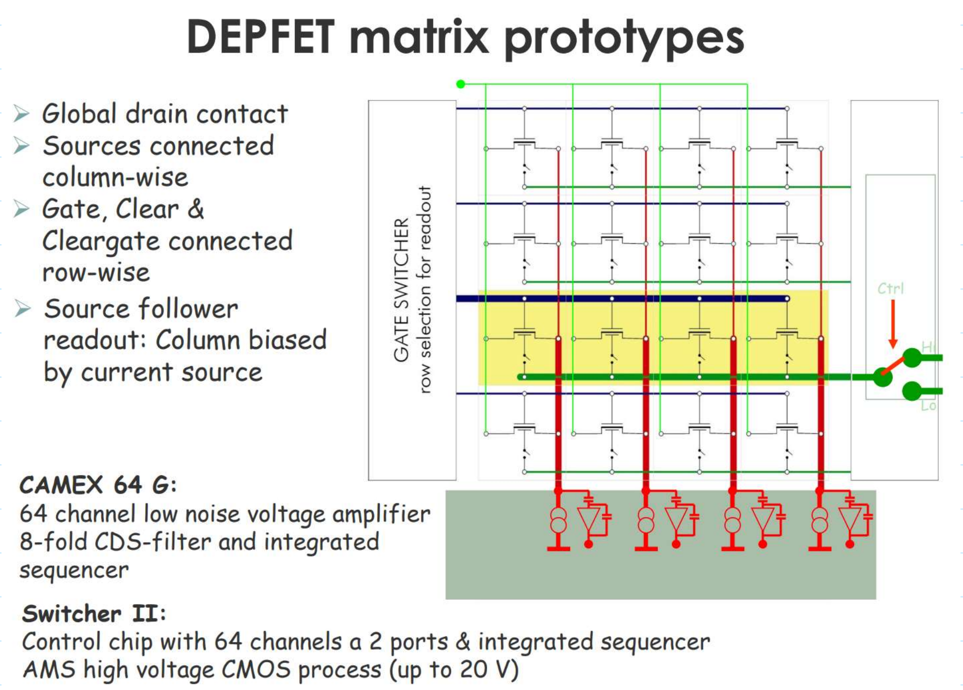
this internal gate is composed on an n+ region



Charge is pushed on n+ internal gate (like in a pin junction)



=> this charge opens or closes the MOSFET channel using FIELD EFFECT

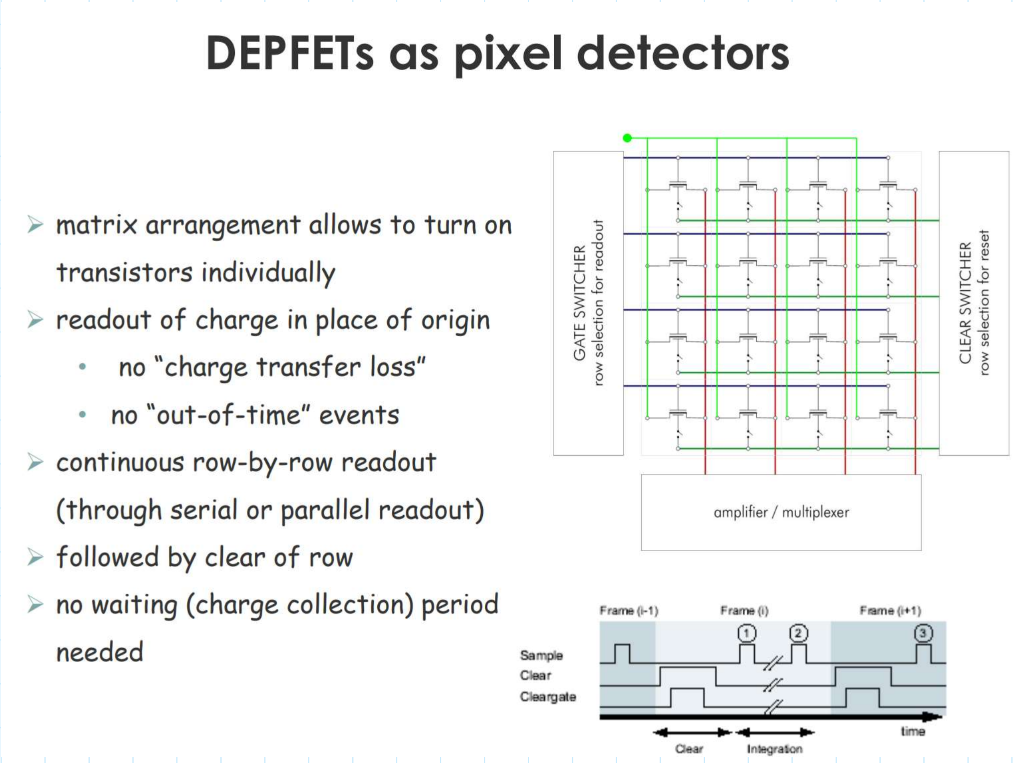


DEPFET matrix prototypes

- Global drain contact
- Sources connected column-wise
- Gate, Clear & Cleargate connected row-wise
- Source follower readout: Column biased by current source

CAMEX 64 G:
64 channel low noise voltage amplifier
8-fold CDS-filter and integrated sequencer

Switcher II:
Control chip with 64 channels a 2 ports & integrated sequencer
AMS high voltage CMOS process (up to 20 V)

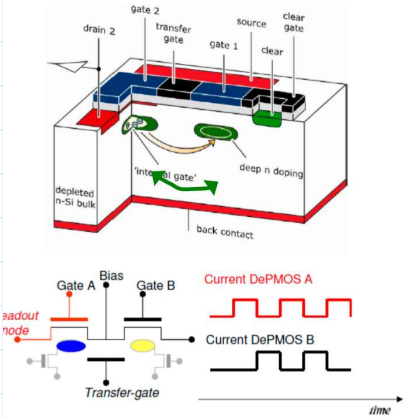


DEPFETs as pixel detectors

- matrix arrangement allows to turn on transistors individually
- readout of charge in place of origin
  - no "charge transfer loss"
  - no "out-of-time" events
- continuous row-by-row readout (through serial or parallel readout)
- followed by clear of row
- no waiting (charge collection) period needed

DEPFET detectors are used in configuration with 1 DEPFET per pixel, as charge can be collected in the internal gate

Multiple readout (ping-pong)



- **Noise reduction technique based by repetitive readout of signal charge**

→ ENC ultimate limitation can be broken

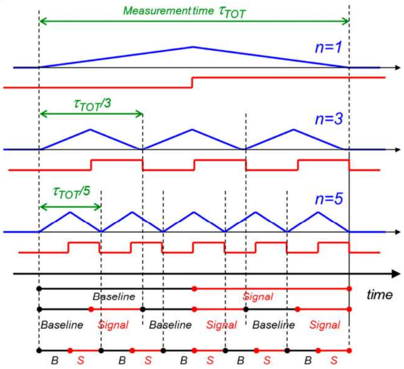
- **Signal charge measured by current difference between filled and empty internal gate**
- **Moving signal charge in and out internal gate N times**
  - noise reduction !
- **We will analyse it for white and 1/f series noise**

The ENC can in principle be lowered below the limit of single-pulse (S/N) analysis

noise analysis

$$ENC^2 = A_1 a C_{tot}^2 \frac{1}{\tau} + A_2(\pi A_f)C_{tot}^2 + A_3(qI_L)\tau$$

p.s. a,b are BILATERAL noise spectral densities. A_f is the 1/f noise coefficient of the single-sided noise spectral density (A_f/f)



Schematic representation of the multiple non-destructive readout with fixed total measurement time τ_{TOT}.

In the upper drawing, only one readout of the signal (red line) is performed, exploiting the whole τ_{TOT}. In the other two cases, the signal is reproduced and read out 3 and 5 times respectively.

The time available for each single measurement is τ_{TOT} /n, number of repetitions.

The reduction of the shaping time for each single measurement (look at the blue weighting functions) turns out in an increased r.m.s. value of the white noise of the individual measurements. The averaging effect of the n readouts (3 and 5 respectively) compensates this noise increment. Therefore, the ENC component related to the white voltage noise does not change with the number of measurements in a fixed time interval.

For the 1/f noise the situation is different. The r.m.s. value of noise of one measurement is independent from the measurement time. This means a single measurement of time length τ_{TOT} /n would result in the same r.m.s. noise.

In this case, the averaging effect of n measurements makes the ENC go down with approximately √n.

white

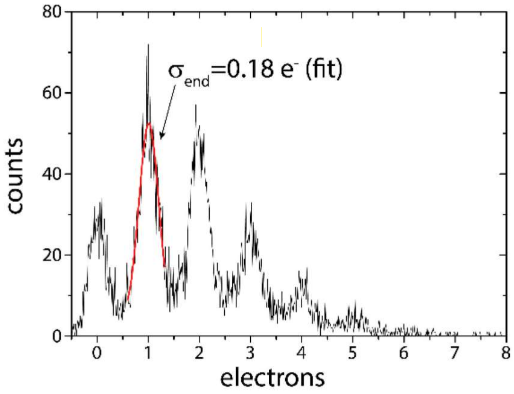
$A_1 \cdots \frac{1}{\tau/N} = ENC_w^2 \cdot N$

averaging

$ENC^2/N \Rightarrow ENC_w^2$ constant

1/f

$ENC^2/N \Rightarrow ENC_{1/f} \downarrow \frac{1}{\sqrt{N}}$



- **sub-electron noise achieved !**
- **increase of ENC for higher no. of readouts most likely related to parallel noise**

→ single photon imaging in the visible !

→ absolute calibration of charge !

Single photon spectrum measured at low light intensity with a circular RNDR-DEPFET at a temperature of −55 °C. Due to the higher amplification the read noise of a single readout is only 3.1 e− rms and a minimum noise of 0.18 e− was obtained with only 300 readouts.